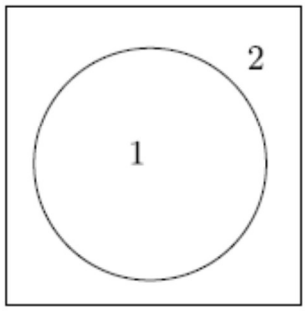
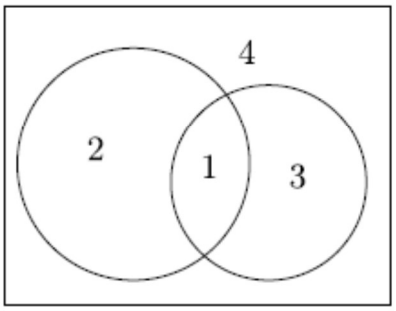
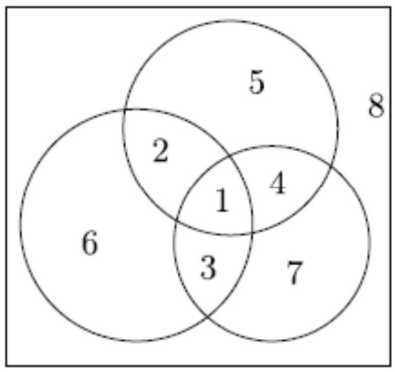
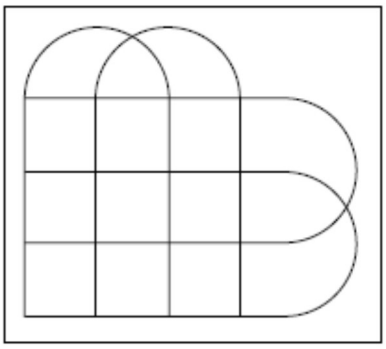
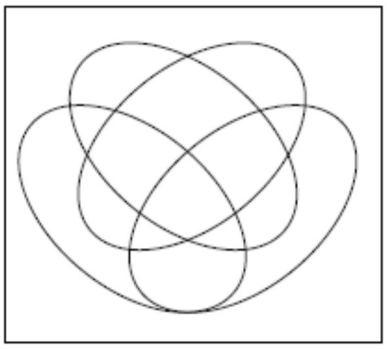


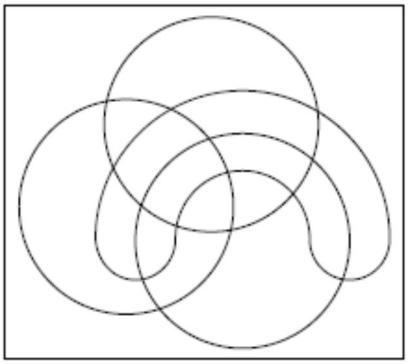



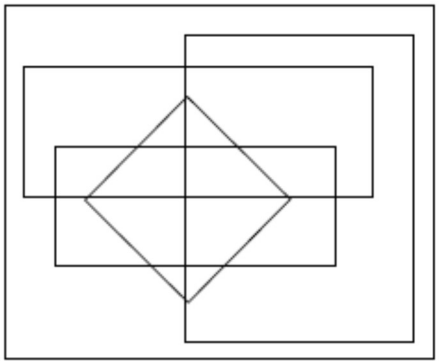



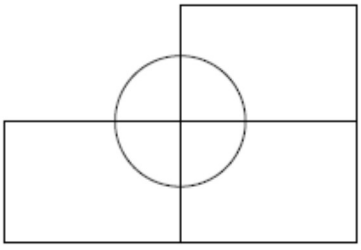

# Venn diagrams

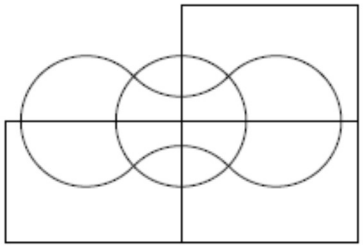


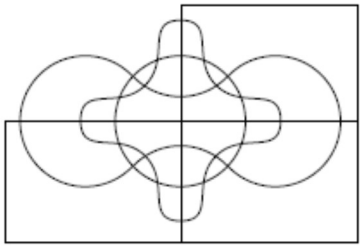










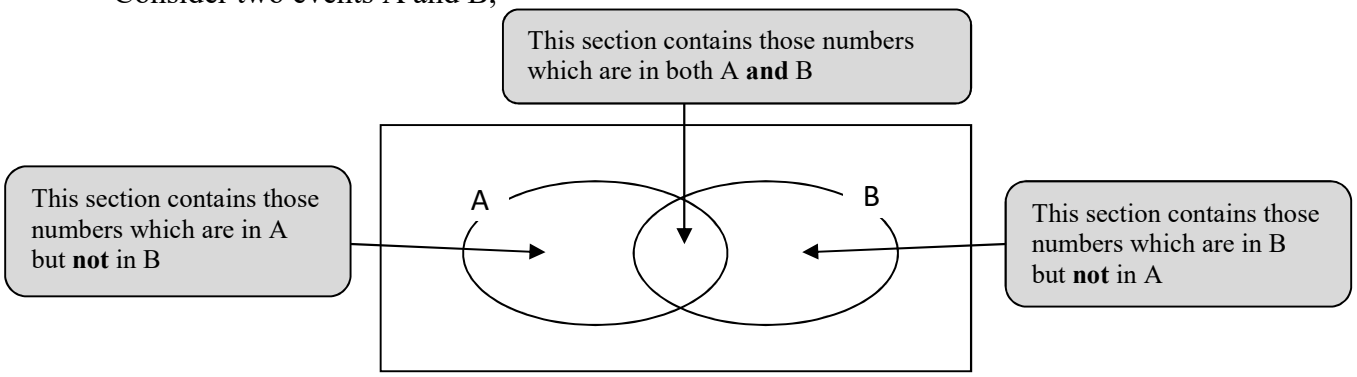


**CALCULATE AND INTERPRET CONDITIONAL PROBABILITIES THROUGH REPRESENTATION USING EXPECTED FREQUENCIES WITH TWO-WAY TABLES, TREE DIAGRAMS AND VENN DIAGRAMS (higher tier)**

## Venn diagrams

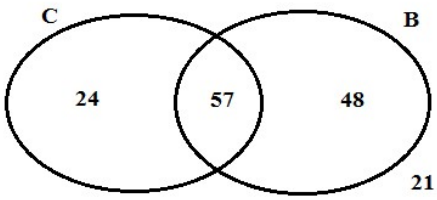
In this section we introduce the ideas of Venn diagrams and probability. A *Venn diagram* is a way of representing information visually.

Consider two events A and B,




### EXAMPLE 1

The Venn diagram shows information of 150 patients in a local surgery.
They were asked if they took any medication for cholesterol (C) or blood pressure (B).




(a) How many patients took both of the medications?
(b) How many patients took only one medication but not both?
(c) What does 21 represent on the Venn diagram?

---

(a) 57 ← Look at the number in the overlap of the circles

(b) 24 + 48 = 72 ← 24 = C only as it is in circle C but not also in circle B; 48 = B only as it is in circle B but not also in circle C

(c) 21 represents the number of patients who took no medication. ← 21 is not in any of the circles

**NOTE:** If you add all the numbers you get the total number of patients.

$24 + 57 + 48 + 21 = 150$

**EXAMPLE 2**

In a group of 30 friends, 15 play netball, 18 play badminton, 7 play netball and badminton.

(a) Represent this information on a Venn diagram.

(b) Work out the probability that a friend plays netball only.

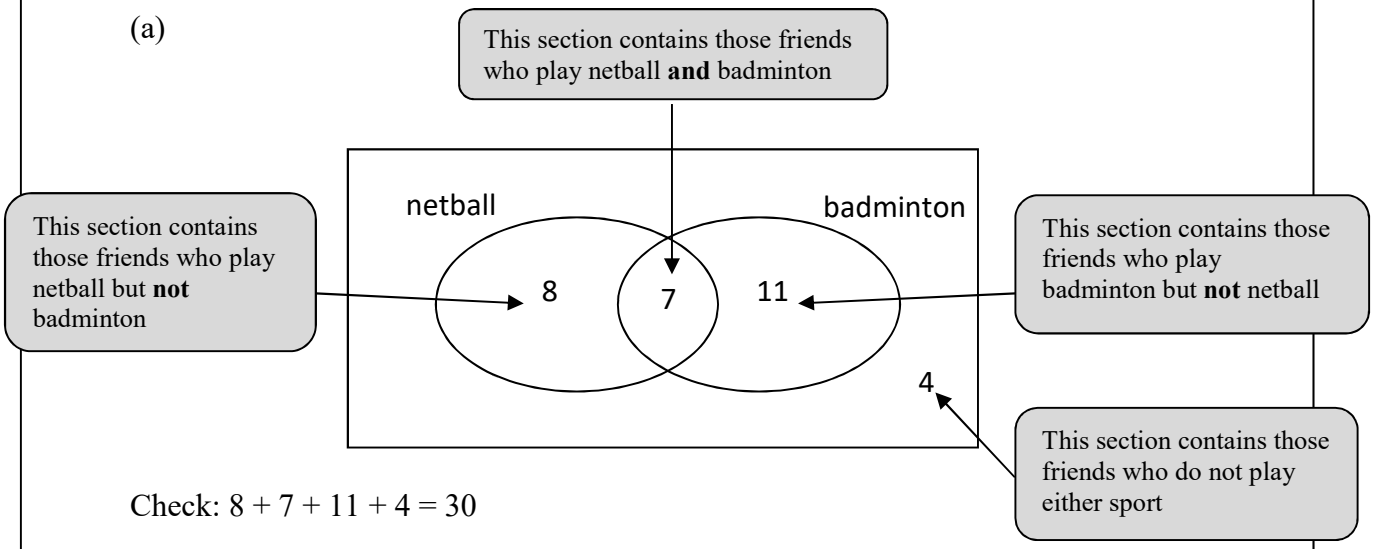
(c) Work out the probability that a friend plays badminton only.

(d) Work out the probability that a friend plays netball or badminton but not both.

(e) Work out the probability that a friend does not play netball or badminton.

---

(a)




Check: $8 + 7 + 11 + 4 = 30$

**NOTE:** Always start from the overlap and then work your way outwards to complete the Venn diagram. Always label your circles.

(b) netball only $= 15 - 7 = 8$ ← 15 play netball but 7 play both

probability netball only $= \frac{8}{30}$ ← 8 play netball only out of 30 friends

(c) badminton only $= 18 - 7 = 11$ ← 18 play netball but 7 play both

probability netball only $= \frac{11}{30}$ ← Write your answer as a fraction

(d) netball only or badminton only $= 8 + 11 = 19$ ← Do not include 7 as 7 play both

probability netball only or badminton only $= \frac{19}{30}$

(e) not netball or badminton $= 30 - (8 + 11 + 7) = 4$ ← 4 is not in any of the circles

probability not netball or badminton $= \frac{4}{30}$

## EXAMPLE 3

The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run, 48 swim, 60 cycle,
40 run and swim, 30 swim and cycle, 35 run and cycle
25 do all three

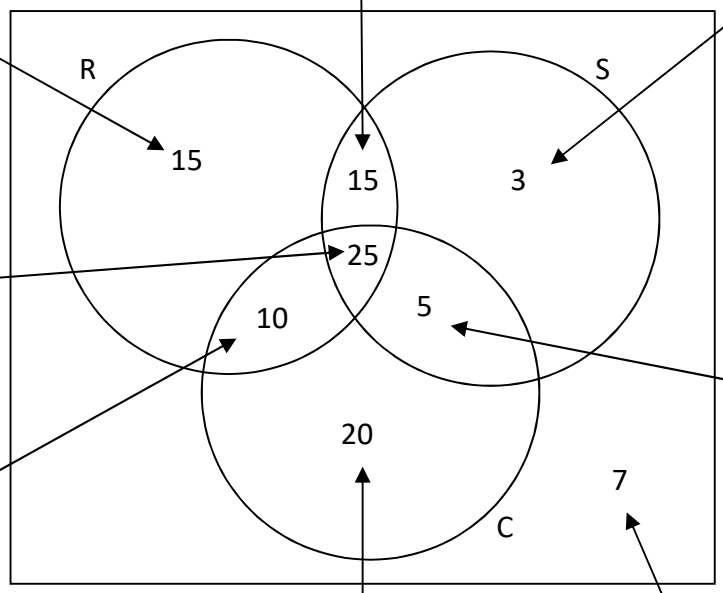
(a) Draw a Venn diagram to represent this information.

(b) Work out the probability that a randomly selected person from the survey

(i) takes none of these types of exercise (ii) swims but does not run

**NOTE:** Always start from the overlap and then work your way outwards to complete the Venn diagram. In this case all of them overlap. We need to draw three overlapping circles. Always label your circles.

(a)




4. 40 run and swim but we already have 25 which run and swim so another 15 run and swim

7. 65 run but we already have 15, 25 and 10 which run so another 15 run
**NOTE** The circle which represents run adds up to 65

6. 48 swim but we already have 15, 25 and 5 which swim so another 3 swim **NOTE** The circle which represents swim adds up to 48

1. All three overlap

3. 30 swim and cycle but we already have 25 which swim and cycle so another 5 swim and cycle

2. 35 run and cycle but we already have 25 which run and cycle so another 10 run and cycle

5. 60 cycle but we already have 10, 25 and 5 which cycle so another 20 cycle
**NOTE** The circle which represents cycle adds up to 60

8. This section represents a person who does not run, swim or cycle so
100 – (15+15+25+10+3+5+20) = 7

(b) (i) none of these types of exercise = 7 ← 7 is not in any of the circles

probability none of these types of exercise = $\frac{7}{100}$ ← There are 100 people in total

(ii) swims but does not run = 3 + 5 = 8 ← In circle S but not in circle R

probability swims but does not run = $\frac{8}{100}$

**EXERCISE 1:**

1. Sandeep gathered some information about the pet dogs and pet cats in his road.
There are 100 families in this road.
68 families have a dog, 42 have a cat and 15 have a dog and a cat.
No family had more than 1 cat or more than 1 dog.

   (a) Draw a Venn diagram to represent this information.
   (b) Find the probability of a family, chosen at random, having neither dog nor a cat.

2. A garage has 50 cars for sale.

   15 of the cars have air conditioning and ABS brakes
   31 of the cars have air conditioning
   17 of the cars have ABS brakes

   Work out the probability of the cars that do **not** have air conditioning or ABS brakes.

3. A running club has 120 members.

   88 of the members take part in road races
   55 of the members take part in marathons
   17 of the members do not run in road races or in marathons

   Work out the probability that a member only takes part in a road race or in a marathon but not both.

4. Anjali asked 60 students in her year group about where they had eaten out in the last month.
Here are her results:

   26 had eaten in Subfood
   11 had eaten in Macdinner and Subfood
   12 had not eaten at Macdinner or Subfood

   (a) Draw a Venn diagram to represent this information.
   (b) Find the probability of a student who had eaten at Macdinner.

5. 90 people in a sports club were surveyed.

   19 play tennis and squash
   50 play tennis
   32 play squash

   (a) Draw a Venn diagram to represent this information.

   One person is chosen at random.
   (b) Work out the probability that
      (i) the person chosen does not play tennis
      (ii) the person chosen plays tennis or squash or both.

6. All the members of a group of 35 students belong to at least one club.
There are 3 clubs: chess, drama and art.

8 of the students belong to only the art club
6 of the students belong to all 3 clubs
3 of the students belong to the chess and art clubs but not to the drama club
18 of the students belong to the art club
3 of the students belong only to the chess club
4 of the students belong only to the drama club

(a) Work out the probability that a student belongs to the chess club and to the drama club but not to the art club?

(b) Work out the probability that a student belongs to the chess club.

7. In a group of 100 students

42 study Statistics
40 study Mathematics
50 study Physics
21 study Mathematics and Physics
19 study Statistics and Physics
17 study Statistics and Mathematics
5 study all three

(a) Draw a Venn diagram to represent this information.

One of the students is picked at random.

(b) Find the probability that this student studies only **one** of these subjects.

8. 140 people were asked in a tasting survey to say which, if any, of three cakes they liked.
Here are the results.

86 people liked cake A
93 people liked cake *B*
76 people liked cake *C*
52 people liked cakes *A* and *B*
51 people liked cakes *B* and *C*
43 people liked cakes *A* and *C*
30 people liked all three cakes.

(a) Draw a Venn diagram to show this information.

A person is chosen at random from those who took part in the survey.

(b) Find the probability that this person
(i) did not like any of the three cakes,
(ii) liked cake *A* but not cake *B*.

9. A group of 200 adults were asked which types of magazines they read.
Their replies showed that

82 read Sports magazines
80 read Garden magazines
84 read Fashion magazines
36 read Sports magazines and read Garden magazines
31 read Sports magazines and read Fashion magazines
25 read Garden magazines and read Fashion magazines
14 read Sports magazines and read Garden magazines and read Fashion magazines

One of the adults asked is to be chosen at random.

Find the probability that this adult
(a) reads none of these types of magazine,
(b) reads exactly two of these types of magazine.

**Conditional probability using Venn diagrams**

**Conditional probability** is the probability of an event occurring **given** that another event has occurred.

For example,

- the probability of David studying GCSE mathematics given that he is studying GCSE physics,
- the probability that I will pay my gas bill given that I have just been paid,
- the probability that my students will turn up to class given that it is a rainy day.

The emphasis is that the probability is influenced by something that has already happened.

P(A | B) means the probability of A occurring, given that B has already occurred.

**EXAMPLE 4**

In a class of 29 girls

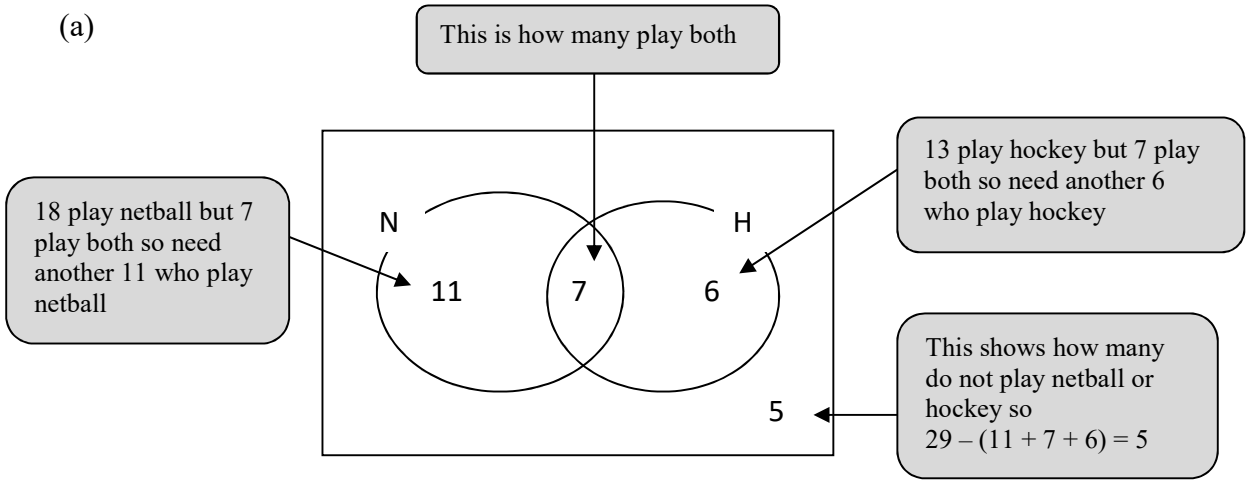
18 girls play netball
13 girls play hockey
7 girls play both

(a) Show this information on a Venn diagram.

One of these girls is picked at random.

(b) Write down the probability that this girl plays hockey given that this girl also plays netball.

(a)




(b) 18 girls play netball ← Given plays netball so look at the netball circle only

7 of these the netball players also play hockey ← We want to know how many of these netball players also play hockey

Probability plays hockey given plays netball = $\frac{7}{18}$

## EXAMPLE 5

In a group of 40 children there are 19 who can swim and 16 who can ride a bike.
There are 5 children who can swim and ride a bike.

A child is selected at random.

(a) Find the probability that this child cannot swim or ride a bike.

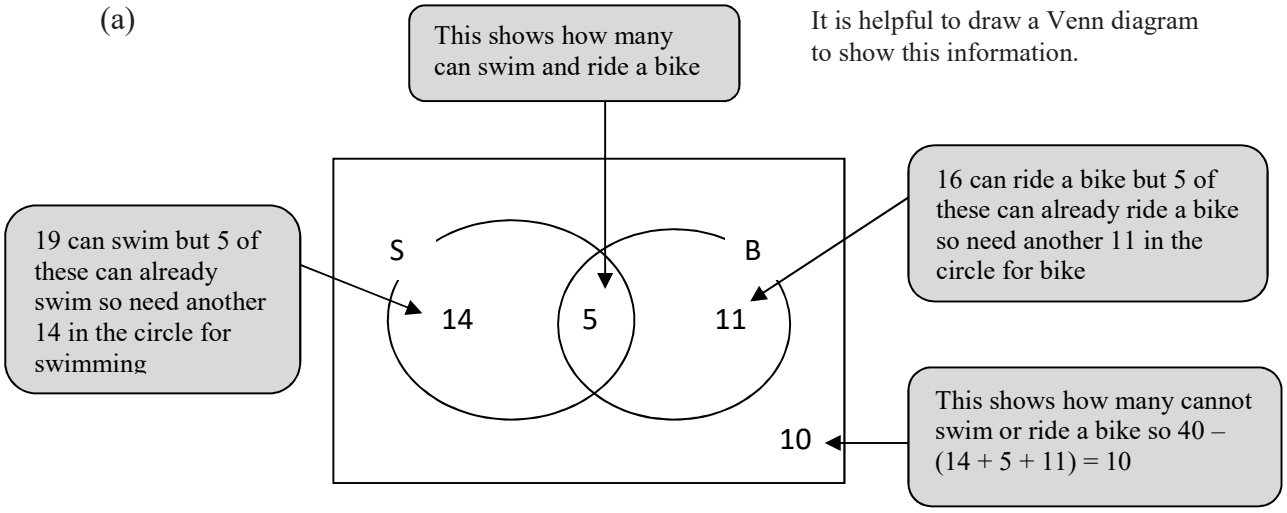
Another child is selected at random.

(b) Given that this child can ride a bike, work out the probability that this child can swim.

---

(a)

It is helpful to draw a Venn diagram to show this information.




Answer: $\frac{10}{40}$ ← 10 of the 40 children cannot swim or ride a bike

**NOTE:** You do not need to cancel the answer to $\frac{1}{4}$

(b) 5 + 11 = 16 ride a bike ← Given rides a bike so look at the bike circle only

5 of these bike riders also swim ← We want to know how many of these bike riders also swim

Probability that a child can swim given rides a bike $= \frac{5}{16}$

## EXAMPLE 6

Charles asked 100 people which of the films Ghost, Titanic and Shrek they have watched. Here is some information about his results.

55 had watched Ghost.
58 had watched Titanic.
60 had watched Shrek.
36 had watched Ghost and Shrek.
35 had watched Ghost and Titanic.
37 had watched Titanic and Shrek.
20 had watched all three films.

(a) Draw a Venn diagram to show this information.

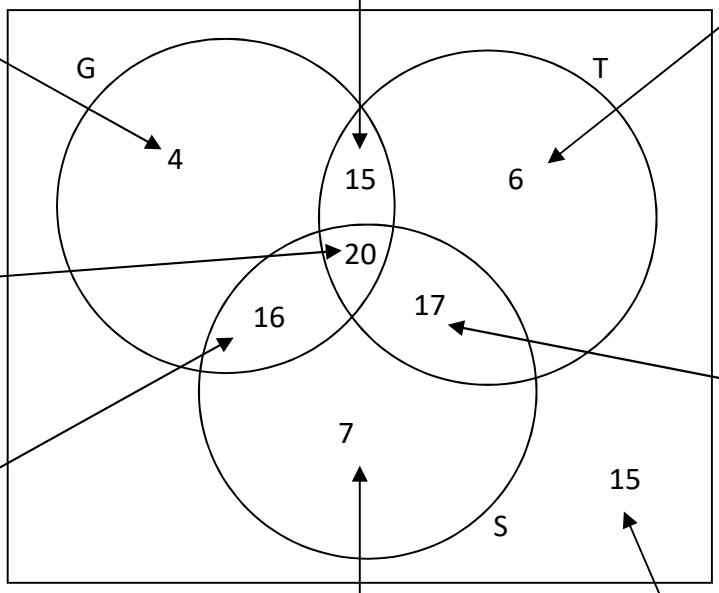
Charles picks, at random, one of these 100 people.

(b) Given that the person had watched Titanic, work out the probability that this person had also watched Ghost.

---

Fill in the numbers on the Venn diagram in the order shown below ... ie starting with 20 watching all 3 films.

(a)




1. All three overlap

2. 37 had watched Titanic and Shrek but we already have 20 who have watched all three so need another 17 who have watched Titanic and Shrek

3. 35 had watched Ghost and Titanic but we already have 20 who have watched all three so need another 15 who have watched Ghost and Titanic

4. 36 had watched Ghost and Shrek but we already have 20 who have watched all three so need another 16 who have watched Ghost and Shrek

5. 60 had watched Shrek but we already have 16, 20 and 17 who have watched Shrek so need another 7 who watch Shrek only.
**NOTE:** The circle which represents Shrek adds up to 60

6. 58 had watched Titanic but we already have 15, 20 and 17 who have watched Titanic so need another 7 who watch Titanic.
**NOTE:** The circle which represents Titanic adds up to 58

7. 55 had watched Ghost but we already have 15, 20 and 16 who have watched Ghost so need another 4 who watch Ghost.
**NOTE:** The circle which represents Ghost adds up to 55

8. This section represents a person who does not run, swim or cycle so
100 – (20+15+16+17+4+6+7) = 15

(b) Number who watched Titanic = 58

← Given watched Titanic so look at the Titanic circle only

Of these 58, 15 + 20 = 35 had also watched Ghost.

← Look at the overlap of G and T

Probability that this person had also watched Ghost = $\frac{35}{58}$

**EXERCISE 2:**

1. 90 children were asked what type of bottled water they took to school.
Their replies are as follows:

52 took sparkling water
36 took still water
14 took both types of water

(a) Show this information on a Venn diagram.

(b) Given that a child takes sparkling water, find the probability that this child also takes still water.

(c) Given that a child takes still water, find the probability that this child also takes sparkling water.

2. In a group of 40 students 6 are left-handed, 18 have size 8 feet and 2 are left-handed with size 8 feet.

(a) Find the probability that a student is left-handed or has size 8 feet,

(b) Given that the student is left-handed, find the probability that a student has size 8 feet.

3. In a survey 100 people were asked whether they watched snooker or cricket when it was on TV. 20 watched neither, 75 watched snooker, 32 watched cricket.

A person is selected at random.

(a) Find the probability that this person watched both cricket and snooker.

(b) Given that this person watched snooker, work out the probability that this person watched cricket.

4. A person's blood group is determined by whether or not it contains any of 3 substances *A*, *B* and *C*. A doctor surveyed 300 patients' blood and produced the table below.

| **Blood contains** | **Number of patients** |
|---|---|
| Only *C* | 100 |
| *A* and *C* but not *B* | 100 |
| Only *A* | 30 |
| *B* and *C* but not *A* | 25 |
| Only *B* | 12 |
| *A*, *B* and *C* | 10 |
| *A* and *B* but not *C* | 3 |

(a) Draw a Venn diagram to show this information.

(b) Find the probability that a randomly chosen patient's blood contains substance *C*.

Harry is one of the patients.

(c) Given that his blood contains substance *A*, find the probability that his blood contains all 3 substances.

5. There are 180 students at a college following a general course in computing.
Students on this course can choose to take up to three extra options.

112 take systems support
70 take developing software
81 take networking
35 take developing software and systems support
28 take networking and developing software
40 take systems support and networking
4 take all three extra options

(a) Draw a Venn diagram to show this information.

A student from the course is chosen at random.

(b) Find the probability that this student takes
(i) none of the three extra options, (ii) networking only.

Students who want to become technicians take systems support and networking.
(c) Given that a randomly chosen student wants to become a technician, find the probability that this student takes all three extra options.

6. 100 people were asked which sports they watched on television.
Here are the results.

36 people watched cricket
28 people watched rugby
36 people watched football
17 people watched both cricket and rugby
19 people watched both cricket and football
15 people watched both rugby and football
10 people watched all three sports

(a) Draw a Venn diagram to show this information.

One of the 100 people is selected at random.

(b) Given that a person watches cricket, find the probability that this person also watches football.
(c) Given that a person watches at least one of the sports, find the probability that this person watches all three.

7. The following shows the results of a juice tasting survey of 100 people.

96 like apple juice
93 like orange juice
96 like mango juice
92 like apple juice and orange juice
91 like orange juice and mango juice
93 like apple juice and mango juice
90 like all three

(a) Draw a Venn diagram to represent this data.

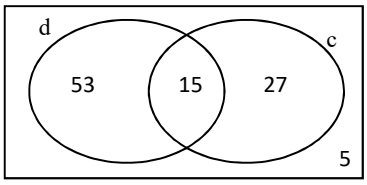
(b) Find the probability that a randomly selected person from the survey likes
(i) none of the three juices, (ii) apple juice but not orange juice.

(c) Given that a person from the survey likes apple juice, find the probability that the person likes mango juice.

**ANSWERS**

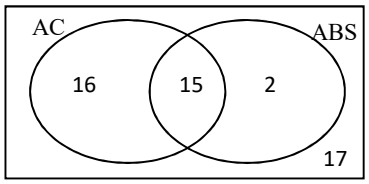
**Exercise 1**

1. (a)



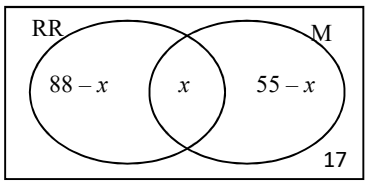

(b) $\frac{5}{100}$

2.




$\frac{17}{50}$

3.

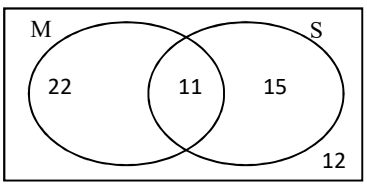



$(88 - x) + (55 - x) + x + 17 = 120$

$x = 40$

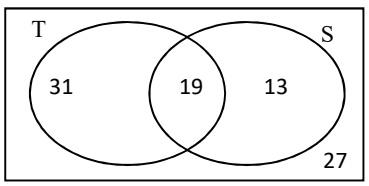
$\frac{48 + 15}{120} = \frac{63}{120}$

4. (a)



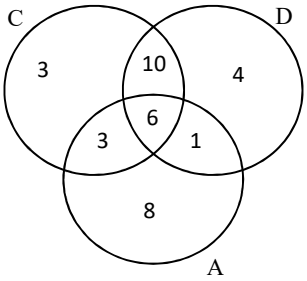

(b) $\frac{22 + 11}{60} = \frac{33}{60}$

5. (a)




(b)(i) $\frac{40}{90}$ (ii) $\frac{63}{90}$

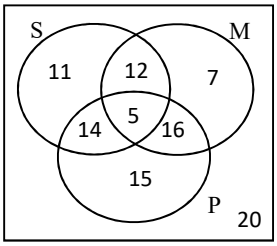
6. (a)




$\frac{10}{35}$

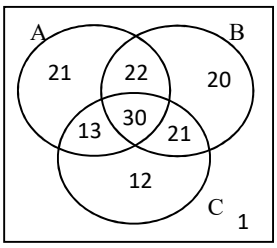
(b) $\frac{22}{35}$

7. (a)



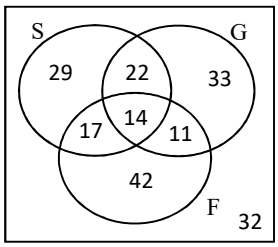

(b) $\frac{11+7+15}{100} = \frac{33}{100}$

8. (a)




(b)(i) $\frac{1}{140}$ (ii) $\frac{21+13}{140} = \frac{34}{140}$

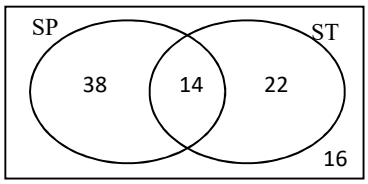
9. (a) $\frac{32}{200}$




(b) $\frac{17+22+11}{200} = \frac{50}{200}$

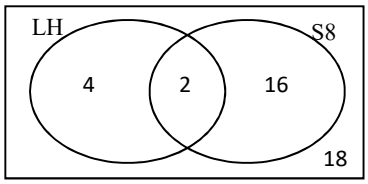
**Exercise 2**

1. (a)



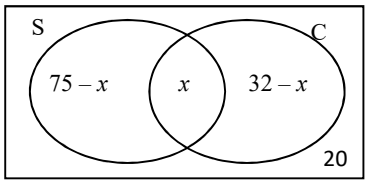

(b) $\frac{14}{52}$ (c) $\frac{14}{36}$

2.




(a) $\frac{20}{40}$ (b) $\frac{2}{6}$

3.

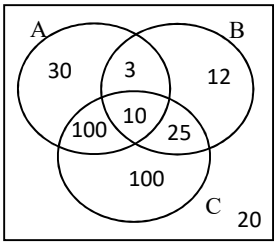



$(75 - x) + x + (32 - x) + 20 = 100$

$x = 27$

(a) $\frac{27}{100}$ (b) $\frac{27}{75}$

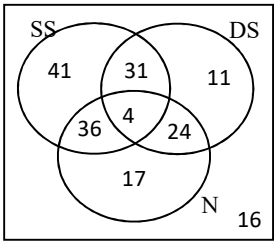
4. (a)




(a) $\frac{100+10+25+100}{300} = \frac{325}{300}$

(b) $\frac{10}{30+100+10+3} = \frac{10}{143}$

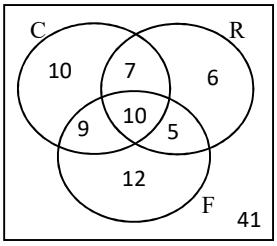
5. (a)




(b)(i) $\frac{16}{180}$ (ii) $\frac{17}{180}$

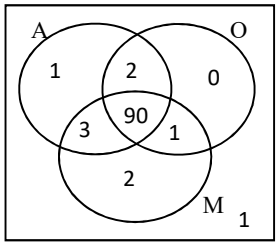
(c) $\frac{4}{40}$

6. (a)




(b) $\frac{19}{36}$ (c) $\frac{10}{59}$

7. (a)




(b)(i) $\frac{1}{100}$ (ii) $\frac{4}{100}$

(c) $\frac{93}{96}$

**Practice Problems**

1. A survey of 230 exotic pet owners shows that:

78 people own tarantulas
81 people own chinchillas
63 people own pot-belly pigs
46 people own chinchillas and tarantulas
62 people own exactly two of these exotic pets

31 people own tarantulas and pot-belly pigs
16 people own tarantulas, chinchillas, and pot-belly pigs
Questions to solve:
a. How many people do not own tarantulas, chinchillas, or pot-belly pigs?
b. How many people own pot-belly pigs?
c. How many people only own chinchillas?

2. A study of 200 books written since 2000 shows that there are three common character types: the villain, the female heroine, and the computer genius.

128 books have a villain or a female heroine
82 books have a female heroine
68 books do not have a female heroine, a villain, or a computer genius
54 books have a villain and a computer genius
28 books have all three character types
49 books have a female heroine and a computer genius
51 books have a villain and a female heroine
Questions to solve:
a. How many books include only a computer genius and a villain?
b. How many books do not have a computer genius?
c. How many books have a female heroine only?

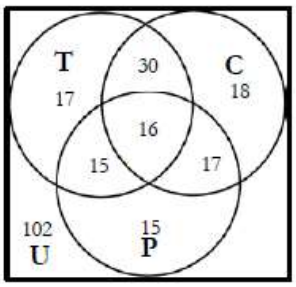
3. A survey of 300 summer movie patrons found that most movie patrons viewed one of three types of movies: comedy, romance, and action.
Let C= represent summer comedies
R= represent summer romance movies
A= represent summer action movies
$n(C)=156$
$n(R)= 106$
$n(A)=133$
$n(C \cap R \cap A)= 8$
$n(C \cap R)=53$
$n(R \cap A)=41$
$n(C \cap A)=87$
Questions to solve:
a. How many summer movie goers did not see a comedy, romance, or action movie?
b. How many summer movie goers saw a comedy or action movie?
c. How many summer movie goers only saw a romance movie?

4. Draw an appropriate Venn diagram, and analyze the known information below.

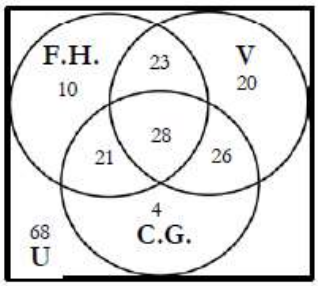
$n(A \cap B)=23$, $n(A \cap B \cap C)=9$, $n(A \cap C)=28$, $n(B \cap C)=11$, $n(A \cap C')=22$
$n(B \cap C')=27$, $n(A' \cap B' \cap C')=17$, $n(C)=37$

Soltions

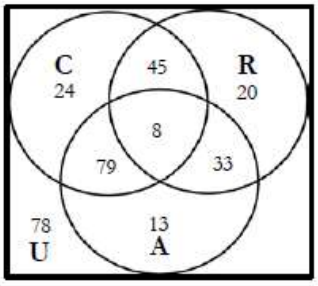
1. a. 102, b. 63, c. 18



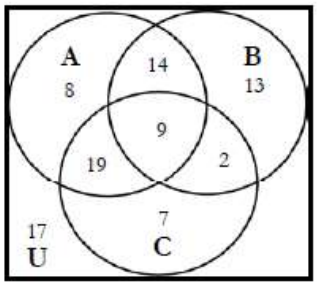

2. a. 26, b. 121, c. 10




3. a. 78, b. 202, c. 20




4.




Sưu Tầm và biên soạn
N.V. LOI