TUYỂN TẬP CÁC BÀI THI TIẾNG ANH GIÀNH CHO HỌC SINH THAM GIA CÁC CUỘC THI TOÁN QUỐC TẾ

HOMC, SINGAPUR, APMOPS, AUSTRAL, TAIWAN, CHINA

1. AMC middle primary - 2016 austral
2. IMC International Mathematics Contest (China), 2017
3. Taiwan IMC International Mathematics Contest , 2012
4. IMSO – 2014
5. APMOPS
6. UK Intermediate Mathematical Challenge - 2O18
7. AMC – 8
8. Austral AMC – 2017
9. SMO Singapore Mathematical Olimpiad – 2014
10. HOMC – Hà nội Open Mthematical Competition - 2016

No.01

AMC MIDDLE PRIMARY (GRADE 5, 6) 2016 AUSTRAL

Câu số 1

Which of these numbers is the smallest?

(A) 655 (B) 566 (C) 565 (D) 555 (E) 556

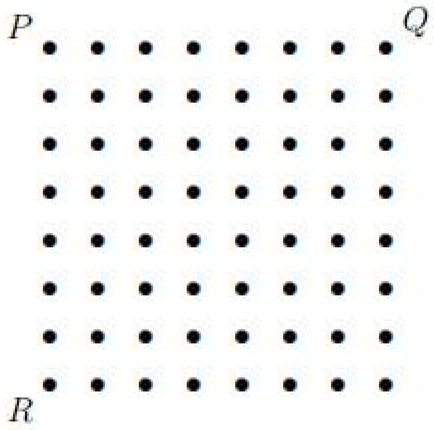
Câu số 2

Two pizzas are sliced into quarters. How many slices will there be?

(A) 2 (B) 10 (C) 6 (D) 8 (E) 16

Câu số 3

Join the dots P, Q, R to form the triangle PQR.



How many dots lie *inside* the triangle PQR?

(A) 13 (B) 14 (C) 15 (D) 17 (E) 18

Câu số 4

$0.3 + 0.4$ is

(A) 0.07 (B) 0.7 (C) 0.12 (D) 0.1 (E) 7

Câu số 5

Lee's favourite chocolates are 80c each. He has five dollars to spend. How many of these chocolates can he buy?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

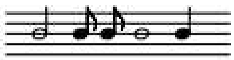
Câu số 6

Ten chairs are equally spaced around a round table. They are numbered 1 to 10 in order. Which chair is opposite chair 9?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Câu số 7

In a piece of music, a note like ♩ is worth one beat, ♪ is worth half a beat, 𝅗𝅥 is worth 2 beats and 𝅝 is worth 4 beats. How many beats are in the following piece of music?



(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

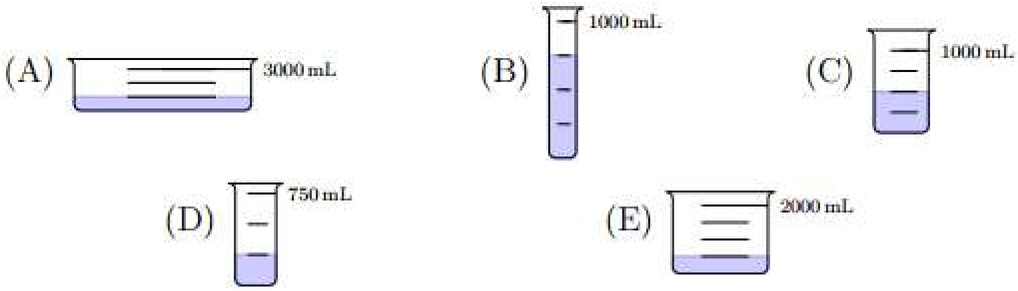
Câu số 8

Phoebe put her hand in her pocket and pulled out 60 cents. How many different ways could this amount be made using 10c, 20c and 50c coins?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

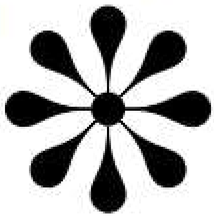
Câu số 9

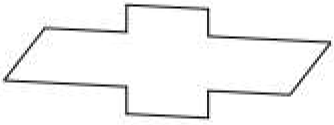
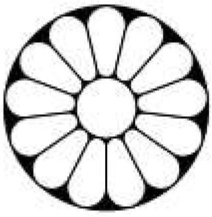
Which of these containers is currently holding the most water?



Câu số 10

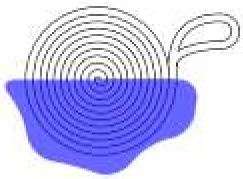
Which of these shapes has the most axes of symmetry (mirror lines)?

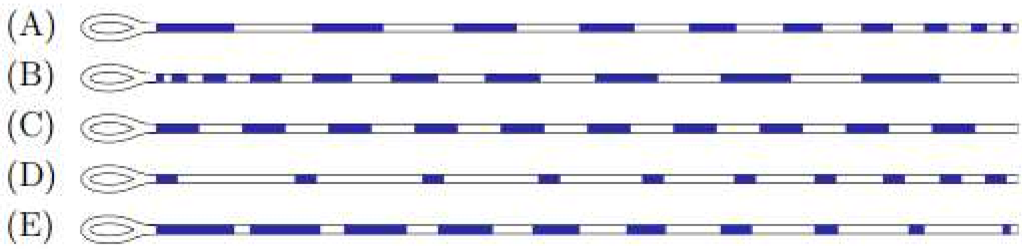
(A)  (B)  (C) 

(D)  (E) 

Câu số 11

A sailor coiled a rope on his ship's deck, and some paint was spilled across half of it. What did the rope look like when it was uncoiled?

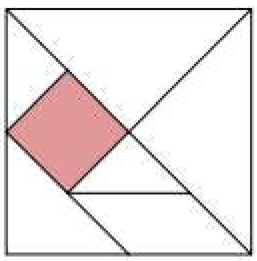




Câu số 12

If the area of the tangram shown is 64 square centimetres, what is the area in square centimetres of the small square?

(A) 32 (B) 24 (C) 16 (D) 8 (E) 4



Câu số 13

For each batch of 25 biscuits, Jack uses $2\frac{1}{2}$ packets of chocolate chips. How many packets does he need if he wants to bake 200 biscuits?

(A) 20 (B) 8 (C) 80 (D) 10 (E) 50

Câu số 14

Which one of the following is correct?

(A) Two even numbers add to an odd number.

(B) An odd number minus an odd number is always odd.

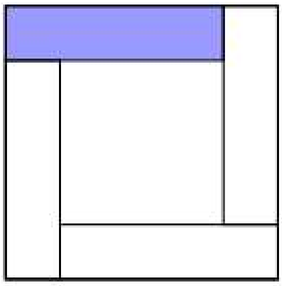
(C) Adding 2 odd numbers and an even number is always odd.

(D) Adding 3 odd numbers is always odd.

(E) An odd number multiplied by an odd number always equals an even number.

Câu số 15

The perimeter of the outer square is 36 cm, and the perimeter of the inner square is 20 cm. If the four rectangles are all identical, what is the perimeter of the shaded rectangle in centimetres?



(A) 12 (B) 14 (C) 24 (D) 20 (E) 18

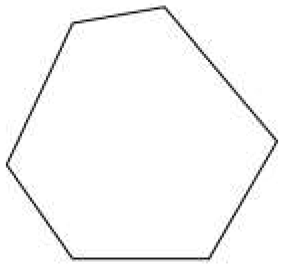
Câu số 16

George has a new lock that opens if the four numbers 1, 2, 3 and 4 are pressed once each in the correct order. If the first number must be larger than the second number, how many combinations are possible?

(A) 10 (B) 12 (C) 15 (D) 18 (E) 20

Câu số 17

A straight cut is made through the hexagon shown to create two new shapes. Which of the following could **not** be made?



(A) one triangle and one hexagon

(B) two pentagons

(C) two quadrilaterals

(D) one quadrilateral and one pentagon

(E) one triangle and one quadrilateral

Câu số 18

The numbers 3, 9, 15, 18, 24 and 29 are divided into two groups of 3 numbers and each group is added. The difference between the two sums (totals) of 3 numbers is as small as possible. What is the smallest difference?

(A) 0 (B) 1 (C) 2 (D) 5 (E) 8

Câu số 19

Benny built a magic square using the numbers from 1 to 16, where the numbers in each row, each column and each diagonal add up to the same total.

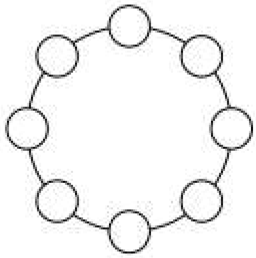
What number does he place at the X?

X			13
5		10	
	7		12
4			1

(A) 16 (B) 15 (C) 17 (D) 11 (E) 14

Câu số 20

Andy has a number of red, green and blue counters. He places eight counters equally spaced around a circle according to the following rules:



- No two red counters will be next to each other.
- No two green counters will be diagonally opposite each other.
- As few blue counters as possible will be used.

How many blue counters will Andy need to use?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Câu số 21

I have five coloured discs in a pile as shown.

I take the top two discs and put them on the bottom (with the red disc still on top of the blue disc).

Then I again take the top two discs and put them on the bottom.

If I do this until I have made a total of 21 moves, which disc will be on the bottom?

(A) red (B) blue (C) green (D) yellow (E) orange

Câu số 22

A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90 kg more than the leopard, and the tiger weighs 50 kg less than the lion. Altogether the three animals weigh 310 kg. How much does the lion weigh?

(A) 180 kg (B) 150 kg (C) 140 kg (D) 130 kg (E) 100 kg

Câu số 23

Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Câu số 24

Jane and Tom are comparing their pocket money. Jane has as many 5c coins as Tom has 10c coins and as many 10c coins as Tom has 20c coins. However, Jane has as many 50c coins as Tom has 5c coins.

They have no other coins and they find that they each have the same amount of money.

What is the smallest number of coins they each can have?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Câu số 25

A tuckshop has two jars of cordial mixture.

Jar A is 30% cordial, while Jar B is 60% cordial.

Some of Jar A is mixed with some of Jar B to make 18 litres of 50% cordial.

How many litres from Jar A are used?

(A) 9 (B) 12 (C) 4

(D) 3 (E) 6

Câu số 26

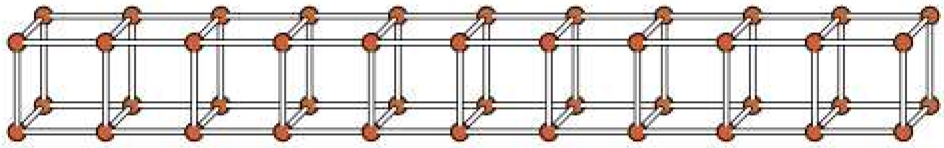
Qiang, Rory and Sophia are each wearing a hat with a number on it. Each adds the two numbers on the other two hats, giving totals of 11, 17 and 22. What is the largest number on a hat?

Câu số 27

The number 840 is the 3-digit number with the most factors. How many factors does it have?

Câu số 28

A class has 2016 matchsticks. Using blobs of modelling clay to join the matches together, they make a long row of cubes. This is how their row starts.

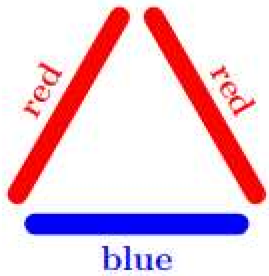


They keep adding cubes to the end of the row until they don't have enough matches left for another cube. How many cubes will they make?

Câu số 29

You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.



Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?

Câu số 30

Today my three cousins multiplied their ages together and it came to 2016. This day last year their ages multiplied to 1377.

When they multiplied their ages together 2 years ago today, what was their answer?

END No.01

No.02

2017年第十三届"IMC国际数学竞赛"（新加坡）

Thirteenth IMC International Mathematics Contest (China), 2017

Grade Six Contest Paper

(August 04 – 07, 2017 Total of 100 points)

A. Multiple Choice Questions. (Each problem is worth 5 marks for a total of 40 marks.)

Khoanh vào câu trả lời đúng. (mỗi câu hỏi được 5 điểm, tổng điểm là 40)

1. What is the simplified value of $\dfrac{9+87+6\times5\times4^3+2\times1}{1+2\times34\times5\times6-7-8-9}$?

Tìm giá trị.

A. $\dfrac{2017}{2018}$ B. 1 C. $\dfrac{2018}{2017}$ D. $\dfrac{6}{5}$

2. The sum of two positive integers is 40 and the greatest common factor of these two integers is 5. What is the difference of these two positive integers?

Tổng của 2 số nguyên dương là 40 và ước chung lớn nhất của 2 số nguyên dương đó là 5. Tính hiệu của hai số nguyên dương.

A. 0 B. 10 C. 30 D. 10 or 30

3. Use the following digits 1, 2, 3, 4, 5, 6, 7, 8 just once to form several composite numbers. What is the smallest possible sum of all these composite numbers?

Sử dụng các chữ số 1, 2, 3, 4, 5, 6, 7, 8, mỗi chữ số đúng một lần để tạo các hợp số (ví dụ một bộ số thoả mãn đề bài là 1234, 56, 78). Tổng nhỏ nhất có thể có của các hợp số đó là bao nhiêu?

A. 90 B. 97 C. 100 D. 140

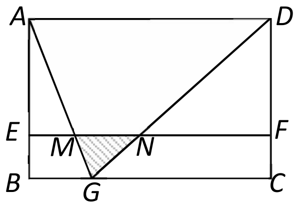
4. In the multiplication puzzle, some digits are given. If this puzzle is established correctly, then what is the sum of all the digits that appear in the □?

Điền các chữ số thích hợp vào phép nhân dưới đây. Tính tổng các chữ số trong các ô vuông được điền.

$$\begin{array}{cccc} & & \square & \square \\ \times & & \square & 2 \\ \hline & \square & 3 & \square \\ \square & 4 & \square & \\ \hline 5 & \square & \square & \square \end{array}$$

A. 28 B. 56 C. 62 D. 70

5. As shown in the diagram, in rectangle *ABCD*, $AE = 2BE$, $DF = 2CF$, *G* lie on *BC*. If area of $\triangle AEM$ is $37\frac{32}{69}\text{cm}^2$, area of $\triangle DFN$ is $62\frac{37}{69}\text{cm}^2$, then what is the area of the shaded region in cm^2?

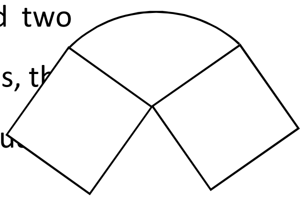


Trong hình bên, hình chữ nhật *ABCD* có *AE=2BE, DF=2CF, G* nằm trên *BC*.

Nếu diện tích của tam giác *AEM* là $37\frac{32}{69}\text{cm}^2$, và diện tích của tam giác *DFN* là $62\frac{37}{69}\text{cm}^2$, thì diện tích của phần bị gạch chéo là bao nhiêu cm^2?

A. 25 B. 37 C. 50 D. 62

6. The given figure is composed of one sector of a circle and two squares, where the arc length of the sector is twice the radius, th perimeter and the area of the given composite figure are equ numerical value. What is the area of the figure in cm^2?



Hình bên tạo bởi 2 hình vuông và 1 phần hình tròn, trong đó chiều dài của phần hình tròn gấp đôi chiều dài của bán kính, chu vi và diện tích của hình bên bằng nhau về giá trị. Tính diện tích của hình bên (bao gồm 2 hình vuông và 1 phần hình tròn) theo cm^2.

A. 2 B. $\frac{8}{3}$ C. 4 D. $21\frac{1}{3}$

7. Alex and Benito, from opposite end of the diameter of a circular runway, started running towards each other. They first met at a certain point that is 80 meters from Benito's starting point. The second time they met at another point that is 60 meters requiring Benito to run for 60 meters in order to be back at the starting point. How long is the runway in meters?

Alex và Benito từ hai đầu của 1 đường kính của 1 vòng chạy hình tròn và bắt đầu chạy về phía nhau. Họ gặp nhau lần đầu ở 1 điểm cách 80m từ chỗ Benito xuất phát. Lần thứ 2 họ gặp nhau tại điểm mà Benito còn phải chạy 60m nữa mới đến điểm xuất phát. Tính chiều dài của một vòng chạy.

A. 200 B. 300 C. 400 D. 500

8. Select four digits from 0, 1, 2, 3, 4, 5 and 6 to form a four-digit distinct numbers so that it is divisible by 45. How many such four-digit numbers are there?

Chọn 4 chữ số từ các chữ số từ 0, 1, 2, 3, 4, 5, 6 để tạo thành các số có 4 chữ số chia hết cho 45. Hỏi có bao nhiêu số có 4 chữ số như vậy?

A. 6 B. 24 C. 28 D. 30

B. Short Answer Questions. (Each problem is worth 5 points for a total of 40 points)

9. Find the simplified value of $\frac{2\times3}{1\times4}+\frac{5\times6}{4\times7}+\frac{8\times9}{7\times10}+\frac{11\times12}{10\times13}+\cdots+\frac{98\times99}{97\times100}$.

Rút gọn:

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10. One summer day, the frog said, "I ate 1210 mosquitoes today." The spider said, "You braggart, I counted and it was only $\overline{a44}$ mosquitoes." The counting is different because the frog having four legs calculated in terms of base 4 while the spider having eight legs calculated in terms of base 8. What is the value of *a*?

Vào một ngày mùa hè, con ếch nói "Tôi đã ăn 1210 con muỗi". Con nhện nói "Bạn nói dối, tôi đã đếm và bạn chỉ ăn có $\overline{a44}$ con thôi". Trên thực tế, cả hai con đều đúng vì con ếch có 4 chân và sử dụng hệ cơ số 4, trong khi con nhện có 8 chân và sử dụng hệ cơ số 8. Giá trị của a trong hệ cơ số 8 là bao nhiêu?

(Một ví dụ về cơ số 5 như sau: số 124 viết trong hệ cơ số 5 sẽ có giá trị trong hệ thập phân là: $1.5^2+2.5+4=39$)

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11. In January, a rack of signature clothes whose selling price of \$300 per piece sold out a total of 40 pieces. In February, the price was cut down by 8%, whereas the sale was increased by 30%. Hence, the profit in February was \$120 more than the profit in January. What is the cost per piece of each clothes?

Vào tháng 1, một cửa hàng bán 1 loại áo với giá 300\$ và bán được 40 chiếc. Vào tháng 2, giá được giảm xuống 8%, và số áo bán được tăng thêm 30%. Số lãi thu được của tháng 2 tăng lên so với tháng 1 là 120\$. Hỏi giá vốn của 1 chiếc áo là bao nhiêu?

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12. A horse ranch has a large stock tank whose bottom has a crack making the water flows out evenly.

Water is filled to the brim of the tank so that 3 horses can drink from it in 8 days; 5 horses in 6 days. How many days can 8 horses drink from the tank when the crack is patched up?

Một trang trại có một bể chứa nước. Bể bị nứt ở dưới đáy nên mỗi ngày có một lượng nước chảy ra như nhau. Khi bể đầy nước thì 3 con ngựa có thể uống trong vừa đúng 8 ngày nhưng 5 con ngựa thì chỉ uống được trong vừa đúng 6 ngày. Nếu vết nứt được bịt lại và bể được đổ đầy nước thì 8 con ngựa sẽ uống hết nước trong bao nhiêu ngày?

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13. In the puzzle: *FOUR* + *FIVE* = *NINE*, each identical letter represents the same digit while different letters represent different digits. What is the least possible four-digit number that *NINE* represents?

Trong phép toán sau: *FOUR* + *FIVE* = *NINE*, mỗi một chữ cái giống nhau biểu diễn một chữ số giống nhau, mỗi chữ cái khác nhau biểu diễn một chữ số khác nhau. Hỏi giá trị bé nhất mà *NINE* có thể nhận được là bao nhiêu?

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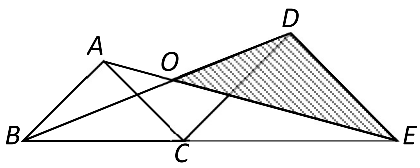
14. In the figure, both $\triangle ABC$ and $\triangle CDE$ are isosceles right-angled triangle with hypotenuse $BC = 7$cm, $CE = 14$cm. It is known that the intersection point of AE and BD is point O, find the area of the shaded region in cm^2.



Trong hình sau, các tam giác *ABC* và *CDE* đều là tam giác vuông cân với cạnh huyền *BC=7cm, CE=14cm*. Biết rằng *O* là giao điểm của *AE* và *BD*. Tìm diện tích của phần bị gạch chéo theo cm^2.

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15. Find the least multi-digit number which contains a string "20" and also a string "17". It is a multiple of 20 and also a multiple of 17.

Tìm số bé nhất sao cho trong số đó có xuất hiện thành phần “20” và “17” và đồng thời số đó chia hết cho 20 và 17.

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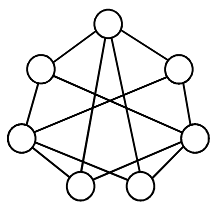
16. Seven circles are connected using lines as shown. Each circle is painted such that every two endpoint circles in each line segment are painted different colors. Using three distinct colors, how many ways can these circles be painted?



7 hình tròn được nối với nhau theo hình vẽ sau. Mỗi hình tròn được tô màu sao cho các hình tròn ở hai đầu của đoạn thẳng được sơn màu khác nhau. Sử dụng 3 màu khác nhau, hỏi có bao nhiêu cách tô màu các hình tròn?

C. Problem Solving. (Each problem is worth 10 points for a total of 20 points. Simplified Solution of each problem is a must and it worth 4 are points)

17. Odd Positive Integers are arranged according to a certain pattern as shown in the table. Arrange all those integers inside the circle in increasing order then (a) what is the 20^{th} integers? (b) what is the sum of the first 20^{th} integers?

41	43	45	47	49	51
39	13	15	17	19	53
37	11	1	3	21	55
35	9	7	5	23	57
33	31	29	27	25	59
				⋯	61

Các số nguyên dương lẻ được sắp xếp theo một quy luật nhất định theo bảng ở hình bên. Sắp xếp tất cả các số nguyên ở trong các hình tròn thành một dãy theo thứ tự tăng dần.

a) Số nguyên thứ 20 trong dãy là số nào?

b) Tổng của 20 số nguyên đầu tiên trong dãy là bao nhiêu?

18. Roy and Sam left City A to City B; and at the same time Terry left City B to City A. After 15 minutes Roy met Terry, and after 1 minute, Sam met Terry. Then, the three of them went traveling between city *A* and city *B*. It is known that Roy's speed is 48 m per min and Sam's speed is 42 m per min. What is the distance (in meters) between Roy and Sam's meeting on the fifth time with Sam and Terry's meeting on the fifth time?

Roy và Sam rời thành phố A đến thành phố B cùng một thời điểm với Terry rời thành phố B đén thành phố A. Sau 15 phút, Roy gặp Terry và 1 phút sau Sam gặp Terry. Sau đó, ba người họ đi qua lại giữa hai thành phố A và B. Biết rằng vận tốc của Roy là 48 m/phút và vận tốc của Sam là 42 m/phút. Tính khoảng cách (đơn vị mét) giữa điểm gặp lần thứ 5 của Roy và Sam và điểm gặp lần thứ 5 của Sam và Terry.

END No.02

No.03

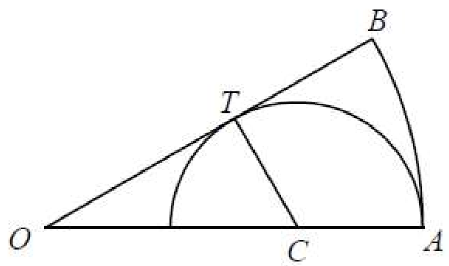
Taiwan International Mathematics Competition 2012

1. In how many ways can 20 identical pencils be distributed among three girls so that each gets at least 1 pencil?

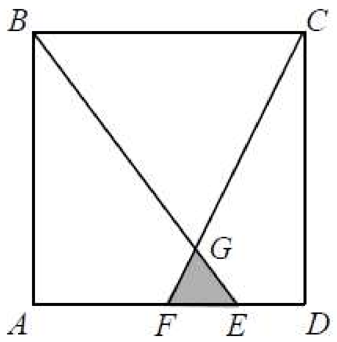
2. On a circular highway, one has to pay toll charges at three places. In clockwise order, they are a bridge which costs \$1 to cross, a tunnel which costs \$3 to pass through, and the dam of a reservoir which costs \$5 to go on top. Starting on the highway between the dam and the bridge, a car goes clockwise and pays toll-charges until the total bill amounts to \$130. How much does it have to pay at the next place if he continues?

3. When a two-digit number is increased by 4, the sum of its digits is equal to half of the sum of the digits of the original number. How many possible values are there for such a two-digit number?

4. In the diagram below, OAB is a circular sector with $OA = OB$ and $\angle AOB = 30°$. A semicircle passing through A is drawn with centre C on OA, touching OB at some point T. What is the ratio of the area of the semicircle to the area of the circular sector OAB?

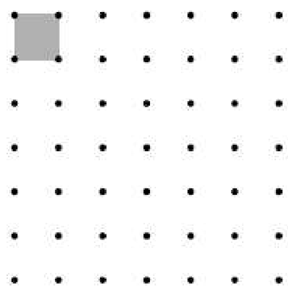


5. *ABCD* is a square with total area 36 cm^2. *F* is the midpoint of *AD* and *E* is the midpoint of *FD*. *BE* and *CF* intersect at *G*. What is the area, in cm^2, of triangle *EFG*?



6. In a village, friendship among girls is mutual. Each girl has either exactly one friend or exactly two friends among themselves. One morning, all girls with two friends wear red hats and the other girls all wear blue hats. It turns out that any two friends wear hats of different colours. In the afternoon, 10 girls change their red hats into blue hats and 12 girls change their blue hats into red hats. Now it turns out that any two friends wear hats of the same colour. How many girls are there in the village? (A girl can only change her hat once.)

7. The diagram below shows a 7×7 grid in which the area of each unit cell (one of which is shaded) is 1 cm^2. Four congruent squares are drawn on this grid. The vertices of each square are chosen among the 49 dots, and two squares may not have any point in common. What is the maximum area, in cm^2, of one of these four squares?



8. The sum of 1006 different positive integers is 1019057. If none of them is greater than 2012, what is the minimum number of these integers which must be odd?

9. The desks in the TAIMC contest room are arranged in a 6×6 configuration. Two contestants are neighbours if they occupy adjacent seats along a row, a column or a diagonal. Thus a contestant in a seat at a corner of the room has 3 neighbours, a contestant in a seat on an edge of the room has 5 neighbours, and a contestant in a seat in the interior of the room has 8 neighbours. After the contest, a contestant gets a prize if at most one neighbour has a score greater than or equal to the score of the contestant. What is maximum number of prize-winners?

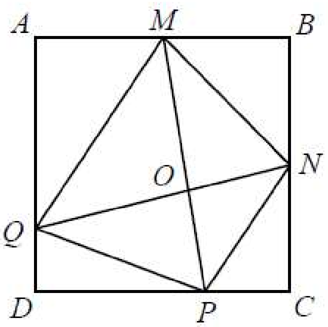
10. The sum of two positive integers is 7 times their difference. The product of the same two numbers is 36 times their difference. What is the larger one of these two numbers?

11. In a competition, every student from school A and from school B is a gold medalist, a silver medalist or a bronze medalist. The number of gold medalist from each school is the same. The ratio of the percentage of students who are gold medalist from school A to that from school B is 5:6. The ratio of the number of silver medalists from school A to that from school B is 9:2. The percentage of students who are silver medalists from both school is 20%. If 50% of the students from school A are bronze medalists, what percentage of the students from school B are gold medalists?

12. We start with the fraction $\frac{5}{6}$. In each move, we can either increase the numerator by 6 or increases the denominator by 5, but not both. What is the minimum number of moves to make the value of the fraction equal to $\frac{5}{6}$ again?

13. Five consecutive two-digit numbers are such that 37 is a divisor of the sum of three of them, and 71 is also a divisor of the sum of three of them. What is the largest of these five numbers?

14. $ABCD$ is a square. M is the midpoint of AB and N is the midpoint of BC. P is a point on CD such that $CP = 4$ cm and $PD = 8$ cm, Q is a point on DA such that $DQ = 3$ cm. O is the point of intersection of MP and NQ. Compare the areas of the two triangles in each of the pairs (QOM, QAM), (MON, MBN), (NOP, NCP) and (POQ, PDQ). In cm^2, what is the maximum value of these four differences?



15. Right before Carol was born, the age of Eric is equal to the sum of the ages of Alice, Ben and Debra, and the average age of the four was 19. In 2010, the age of Debra was 8 more than the sum of the ages of Ben and Carol, and the average age of the five was 35.2. In 2012, the average age of Ben, Carol, Debra and Eric is 39.5. What is the age of Ben in 2012?

END No.03

No. 04

International Mathematics and Science Olympiad 2014

SHORT ANSWER PROBLEMS

(1) Calculate 9+99+999+9999+99999+999999+9999999+99999999+999999999.

(2) The income of a taxi driver is the sum of the regular salary and some tips. The tips are 5/4 of his salary. What is the fraction of his income which comes from his tips?

(3) The base of a large triangle is two times the altitude of a small triangle, and the altitude of the large triangle is three times the base of the small triangle. What is the ratio of the area of the large triangle to the area of the small triangle?

(4) In an election, only 80% of people planned to vote. However only 85% of those who planned to vote actually vote. What is the percentage of the people who actually vote?

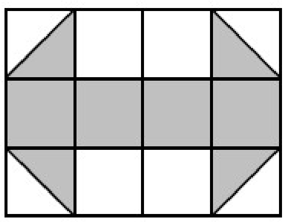
(5) INTERNUTS company offers internet service with an initial payment of 300000 rupiahs and a monthly fee of 72000 rupiahs. Another company, VIDIOTS, offers internet service with no initial payment but a monthly fee of 90000 rupiahs. Johnny prefers INTERNUTS company. What is the minimum number of months he should subscribe in order to pay less than the subscription with VIDIOTS company?

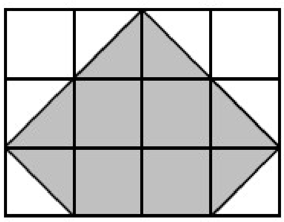
(6) Twelve identical squares are put together in a 6 × 2 configuration to form a rectangle. If the perimeter of each square is 6 cm, what is the perimeter of the rectangle?

(7) The sum of two 2-digit numbers is also a 2-digit number. What is the maximum value of the product of those three 2-digit numbers?

(8) How many positive integers less than 2014 such that the sum of the digits of each is a multiple of 5?

(9) The diagram below shows two 3 × 4 pieces of paper, part of which is shaded. A student copies both figures on the same 3 × 4 piece of paper. What is the fraction of this piece of paper which is shaded?





(10) In total: Alice and Brian have 377 cards, Brian and Colin have 685 cards, Colin and Alice have 546 cards. How many cards does Brian have?

(11) Write down six positive integers whose sum is 100 such that each integer contains at least one digit 6.

(12) *ABCD* is a square piece of paper. *M* and *N* are the respective midpoints of *AB* and *CD*. *P* is a point on *AM* such that if the piece of paper is folded along *DP*, then *A* lands on a point *Q* on the segment *MN*. What is the degree of $\angle ADP$?

(13) The sum of five different positive integers is 364, and the largest one is 95. What is the maximum possible value of the smallest integer of these five integers?

(14) Alice gives 1/4 of her apples to Brian and 1/3 of the remaining apples toColin. The leftover apples of hers are worth 13500 rupiahs. What is the worth of Colin's apples received from Alice?

(15) *ABCD* is a rectangle with *AD* = 8 cm and *CD* = 12 cm. *P* is the point on *CD* such that *DP* = *AD* and *Q* is the point on *AD* such that *DQ* = *CP*. What is the area of the quadrilateral *ABPQ*?

(16) Each pair of five positive integers is added, so there are ten sums, which are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. What is the largest integer among these five integers?

(17) What is the sum of all multiples of 6 each of which has exactly ten positive divisors?

(18) What is the smallest positive integer which leaves remainders of 3, 4 and 5 when divided respectively by 5, 7 and 9?

(19) What is the remainder when $1 \times 2 \times 3 \times \cdots \times 14 \times 15$ is divided by $1 + 2 + 3 + \cdots + 14 + 15$?

(20) We want to draw a number of straight lines such that for each square of a chessboard, at least one of the lines passes through an interior point of the square. At least how many lines we need for a 3×3 chessboard?

(21) Three children go on an 84 km trip. Each can walk at 5 km/h or ride a bicycle at 20 km/h, but they only have two bicycles among them. At any moment, a bicycle can take only one rider. Also, bicycles can safely be left on the roadside. At least how many hours for all three children to finish the trip?

(22) An ant crawls along the surface of a $3 \times 3 \times 3$ cube from one corner to the farthest corner. It must travel along exactly 9 unit segments on the edges of the cube or on the faces of the cube separating two 1×1 squares. How many such routes are there?

(23) Andy writes a positive integer using each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once. Any sequence of two adjacent digits in it forms an integer which is divisible by either 7 or 13. Write down all possible integers that Andy could write.

(24) In triangle *ABC*, the bisector of $\angle B$ intersects *CA* at *E* and the bisector of $\angle C$ intersect *AB* at *F*. If $\angle BEF = 24°$ and $\angle CFE = 18°$, what is the degree of $\angle CAB$?

(25) *ABC* is a triangle such that *AB* = 1 cm and *BC* = 1.5 cm. *D* is a point on the line through *A* parallel to *BC*, such that *CD* = 4 cm. Write down all possible integral values of the length of *AD*.

END No.04.

No. 05.

Singapore-Asia Pacific Mathematical Olympiad for Primary Schools (Mock Test for APMOPS 2012)

1 Suppose that today is Tuesday. What day of the week will it bee 100 days from now?

2 Ariel purchased a certain amount of apricots. 90% of the apricot weight was water. She dried the apricots until just 60% of the apricot weight was water. 15 kg of water was lost in the process. What was the original weight of the apricots (in kg)?

3 Notice that $1-2=-1$, $1-(2-3)=2$, and $1-(2-(3-4))=-2$. What is the value of

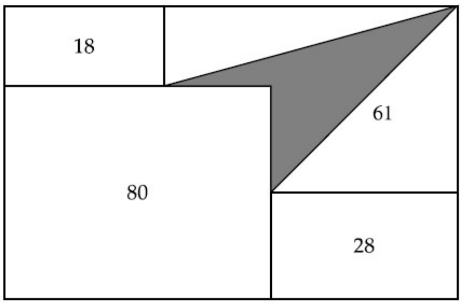
$$1-(2-(3-(4-\ldots-100)))\ldots)?$$

4 You are preparing skewers of meatballs, where each skewer has either 4 or 6 meatballs on it. Altogether you use 32 skewers and 150 meatballs. How many skewers have only 4 meatballs on them?

5 The numbers $1, 2, 3, \ldots, 100$ are written in a row. We first remove the first number and every second number after that. With the remaining numbers, we again remove the first number and every second number after that. We repeat this process until one number remains. What is this number?

6 P and Q are whole numbers so that the ratio $P : Q$ is equal to $2 : 3$. If you add 100, 200 to each of P and Q, the new ratio becomes equal to $3 : 4$. What is P?

7 The following figure consists of 3 smaller rectangles and a hexagon whose areas are 18 cm^2, 80 cm^2, 28 cm^2, and 61 cm^2. If all the side-lengths in centimetre of the rectangles and the hexagon are integers, find the area of the shaded region.



8 Suppose that a, and b are positive integers, and the four numbers

$$a+b,\ a-b,\ a\times b,\ a\div b$$

are different and are all positive integers. What is the smallest possible value of $a+b$?

9 You are given a two-digit positive integer. If you reverse the digits of your number, the result is a number which is 20% larger than your number. What is your number?

10 The number $N = 111\ldots1$ consists of 2006 ones. It is exactly divisible by 11. How many zeros are there in the quotient $\frac{N}{11}$?

11 This mock test, prepared by **Binh** from **HDS** centre in Hanoi, consists of 30 problems. Pupil A gets a score that is an odd multiple of 5 and pupil B gets a score that is a even multiple of 7. The mark of each problem is an integer and each of the two pupils' score is an integer the difference of which is 3 and sum is less than 100. Find the higher score of the pupils.

12 In a large hospital with several operating rooms, ten people are each waiting for a 45 minute operation. The first operation starts at 8:00 a.m., the second at 8:15 a.m., and each of the other operations starts at 15 minute intervals thereafter. When does the last operation end?

13 Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

14 A work crew of 3 people requires 20 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each of both crews works at the same rate as each of the others?

15 Each of the nine numbers $1, 2, \ldots, 9$ is to be placed inside the cell of the following 3×3 grid once. The product of three numbers in each row and in each column is given: the product of numbers in the first column is 35, the second column is 96, the product of three numbers in the first row is 54, etc. Find the value of $p + q$.

35	96	108	
			54
	p		42
		q	160

16 N is a whole number greater than 2. The six faces of a $5 \times 5 \times N$ block of wood are painted red and then the block cut into $25\ N1 \times 1 \times 1$unit cubes. If exactly 92 unit cubes have exactly two faces painted red, what is N?

17 Eleven people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. The total number of handshakes that occurs is x. Find the value of x

18 Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. The number of white marbles that Mark adds to the bag is n. Find the value of n.

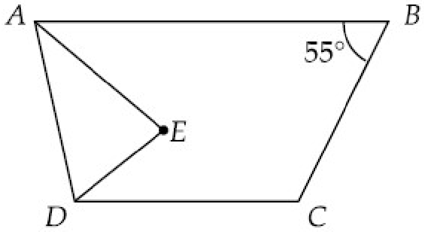
19 A number line has 40 consecutive integers marked on it. If the smallest of these integers is ˇ11, what is the largest?

20 If I add 5 to $\frac{1}{3}$ of the number, the result is $\frac{1}{2}$ of the number. What is the number?

21 If n is a positive inter such that all the following numbers are prime, find the value of n.

$$5n - 7,\ 3n - 4,\ 7n + 3,\ 6n + 1,\ 9n + 5.$$

22 The diagram below shows a trapezium with base AB and CD, $\angle ABC = 55°$. E is inside the trapezium such that AE bisects angle BAD and ED bisects angle ADC. If the measure of $\angle DAE$ is $x°$, find the value of x.



23 Let $\lfloor a \rfloor$ denote the integer not exceeding a. If n is a whole number, $n \geq 2$, find $\lfloor p \rfloor$

$$p = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^3} + \cdots + \frac{n}{2^n}.$$

24 How many numbers are there that appear both in the arithmetic sequence 10, 16, 22, 28, ..., 1000 and the arithmetic sequence 10, 21, 32, 43, ..., 1000?

25 The whole numbers from the set $\{1, 2, 3, \ldots, 2020\}$ are arranged in the following manner.

1					
2	3				
4	5	6			
7	8	9	10		
11	12	13	14		
15	16	17	18	19	
20	21	22	23	24	25
⋮					

What is the number that will appear directly below the number 2012?

Answer Key

Question	Answer	Question	Answer	Question	Answer
1	Thursday	11	28	21	30
2	20	12	11AM	22	90°
3	−50	13	21	23	1
4	21	14	15	24	16
5	64	15	6	25	2075
6	200	16	19	26	50
7	20	17	55	27	200 m^2
8	8	18	4	28	$\frac{1}{10}$
9	45	19	28	29	2000π
10	1002	20	30	30	29381654729

END No.05

No.06

UK Intermediate Mathematical Challenge

THURSDAY 1ST FEBRUARY 2018

1. Which of these is the sum of the cubes of two consecutive integers?

A 4   B 9   C 16   D 25   E 36

2. How many of these four integers are prime?

1   11   111   1111

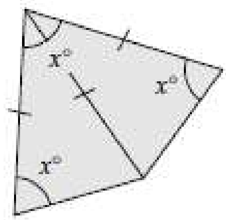
A 0   B 1   C 2   D 3   E 4

3. In September 2016 a polymer £5 note was introduced. The Bank of England issued 440 million of them.
What is the total face value of all these notes?

A £220 000 000   B £440 000 000   C £2 200 000 000   D £4 400 000 000   E £22 000 000 000

4. A kite is made by joining two congruent isosceles triangles, as shown.
What is the value of x?

A 36   B 54   C 60   D 72   E 80



5. The adult human body has 206 bones. Each foot has 26 bones.
Approximately what fraction of the number of bones in the human body is found in one foot?

A $\frac{1}{6}$ B $\frac{1}{8}$ C $\frac{1}{10}$ D $\frac{1}{12}$ E $\frac{1}{20}$

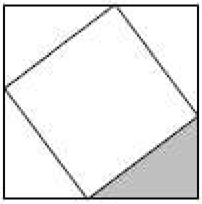
6. In 2014, in Boston, Massachusetts, Eli Bishop set a world record for the greatest number of claps per minute. He achieved 1020 claps in one minute.
How many claps is that per second?

A 17   B 16.5   C 16   D 15.5   E 15

7. How many two-digit squares have the property that the product of their digits is also a square?

A 0 B 1 C 2 D 3 E 4

8. The diagram shows a square of perimeter 20 cm inscribed inside a square of perimeter 28 cm.
What is the area of the shaded triangle?



A 6 cm^2 B 7 cm^2 C 8 cm^2 D 9 cm^2 E 10 cm^2

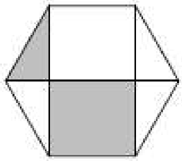
9. Which integer n satisfies $\frac{3}{10} < \frac{n}{20} < \frac{2}{5}$?

A 3 B 4 C 5 D 6 E 7

10. Which of these integers cannot be expressed as the difference of two squares?

A 5 B 7 C 8 D 9 E 10

11. The diagram shows a regular hexagon which has been divided into six regions by three of its diagonals. Two of these regions have been shaded. The total shaded area is 20 cm^2.
What is the area of the hexagon?



A 40 cm^2 B 48 cm^2 C 52 cm^2 D 54 cm^2 E 60 cm^2

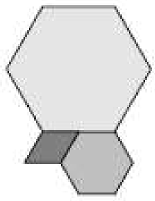
12. Someone has switched the numbers around on Harry's calculator!
The numbers should be in the positions shown in the left-hand diagram, but have been switched to the positions in the right-hand diagram.
Which of the following calculations will *not* give the correct answer when Harry uses his calculator?

7	8	9
4	5	6
1	2	3

9	8	7
6	5	4
3	2	1

A 79×97 B 78×98 C 147×369 D 123×321 E 159×951

13. The diagram shows a rhombus and two sizes of regular hexagon.
What is the ratio of the area of the smaller hexagon to the area of the larger hexagon?



A 1 : 2 B 1 : 3 C 1 : 4 D 1 : 8 E 1 : 9

14. Which of these is equal to $\frac{10}{9} + \frac{9}{10}$?

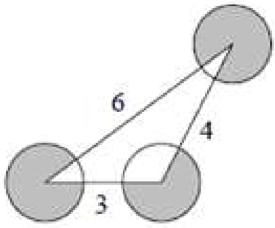
A 1 B 2 C $2.0\dot{1}$ D $2.\dot{1}$ E $2.\dot{2}$

15. How many of these four shapes could be the shape of the region where two triangles overlap?

equilateral triangle   square   regular pentagon   regular hexagon

A 0   B 1   C 2   D 3   E 4

16. The diagram shows a triangle with edges of length 3, 4 and 6.
A circle of radius 1 is drawn at each vertex of the triangle.
What is the total shaded area?

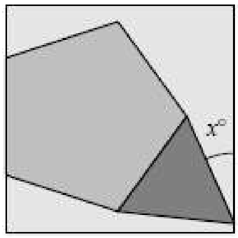


A 2π B $\frac{9\pi}{4}$ C $\frac{5\pi}{2}$ D $\frac{11\pi}{4}$ E 3π

17. How many three-digit numbers are increased by 99 when their digits are reversed?

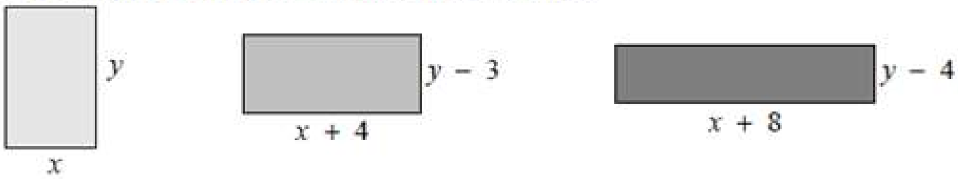
A 4   B 8   C 10   D 80   E 90

18. The diagram shows a regular pentagon and an equilateral triangle placed inside a square.
What is the value of x?



A 24   B 26   C 28   D 30   E 32

19. The three rectangles shown below all have the same area.



What is the value of $x + y$?

A 4   B 6   C 8   D 10   E 12

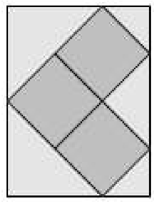
20. A particular integer is the smallest multiple of 72, each of whose digits is either 0 or 1.
How many digits does this integer have?

A 4   B 6   C 8   D 10   E 12

21. For certain values of x, the list x, $x + 6$ and x^2 contains just two different numbers. How many such values of x are there?

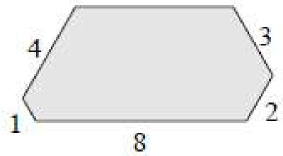
A 1 B 2 C 3 D 4 E 5

22. Three squares, with side-lengths 2, are placed together edge-to-edge to make an L-shape. The L-shape is placed inside a rectangle so that all five vertices of the L-shape lie on the rectangle, one of them at the midpoint of an edge, as shown.
What is the area of the rectangle?



A 16 B 18 C 20 D 22 E 24

23. The diagram shows a hexagon. All the interior angles of the hexagon are 120°. The lengths of some of the sides are indicated.
What is the area of the hexagon?

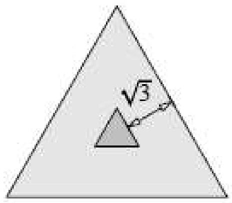


A $20\sqrt{3}$ B $21\sqrt{3}$ C $22\sqrt{3}$ D $23\sqrt{3}$ E $24\sqrt{3}$

24. A list of 5 positive integers has mean 5, mode 5, median 5 and range 5.
How many such lists of 5 positive integers are there?

A 1 B 2 C 3 D 4 E 5

25. The diagram shows two equilateral triangles. The distance from each point of the smaller triangle to the nearest point of the larger triangle is $\sqrt{3}$, as shown.
What is the difference between the lengths of the edges of the two triangles?



A $2\sqrt{3}$ B $4\frac{1}{2}$ C $3\sqrt{3}$ D 6 E $4\sqrt{3}$

END. No6

No. 07.

AMC - 8

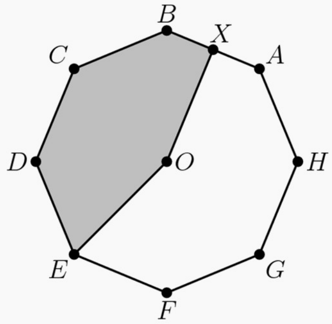
Problem 1

How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)

$\textbf{(A) }12\qquad\textbf{(B) }36\qquad\textbf{(C) }108\qquad\textbf{(D) }324\qquad\textbf{(E) }972$

Problem 2

Point O is the center of the regular octagon $ABCDEFGH$, and X is the midpoint of the side $\overline{AB}$. What fraction of the area of the octagon is shaded?



$\textbf{(A) }\frac{11}{32}\qquad\textbf{(B) }\frac{3}{8}\qquad\textbf{(C) }\frac{13}{32}\qquad\textbf{(D) }\frac{7}{16}\qquad\textbf{(E) }\frac{15}{32}$

Problem 3

Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

$\textbf{(A) }5\qquad\textbf{(B) }6\qquad\textbf{(C) }8\qquad\textbf{(D) }9\qquad\textbf{(E) }10$

Problem 4

The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?

$\textbf{(A) }2\qquad\textbf{(B) }4\qquad\textbf{(C) }5\qquad\textbf{(D) }6\qquad\textbf{(E) }12$

Problem 5

Billy's basketball team scored the following points over the course of the first 11 games of the season:

$$42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73$$

If his team scores 40 in the 12th game, which of the following statistics will show an increase?

$\textbf{(A) }\text{range}\qquad\textbf{(B) }\text{median}\qquad\textbf{(C) }\text{mean}\qquad\textbf{(D) }\text{mode}\qquad\textbf{(E) }\text{mid-range}$

Problem 6

In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 420 \qquad \textbf{(C)}\ 500 \qquad \textbf{(D)}\ 609 \qquad \textbf{(E)}\ 701$

Problem 7

Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{2}{9} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{5}{9}$

Problem 8

What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 29 \qquad \textbf{(C)}\ 43 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 57$

Problem 9

On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20days?

$\textbf{(A)}\ 39 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 210 \qquad \textbf{(D)}\ 400 \qquad \textbf{(E)}\ 401$

Problem 10

How many integers between 1000 and 9999 have four distinct digits?

$\textbf{(A)}\ 3024 \qquad \textbf{(B)}\ 4536 \qquad \textbf{(C)}\ 5040 \qquad \textbf{(D)}\ 6480 \qquad \textbf{(E)}\ 6561$

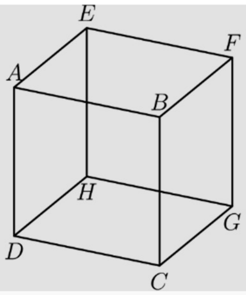
Problem 11

In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel ($A, E, I, O,$ or U), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "$AMC8$"?

$\textbf{(A)}\ \frac{1}{22,050} \qquad \textbf{(B)}\ \frac{1}{21,000} \qquad \textbf{(C)}\ \frac{1}{10,500} \qquad \textbf{(D)}\ \frac{1}{2,100} \qquad \textbf{(E)}\ \frac{1}{1,050}$

Problem 12

How many pairs of parallel edges, such as $\overline{AB}$ and $\overline{GH}$ or $\overline{EH}$ and $\overline{FG}$, does a cube have?



$\textbf{(A) } 6 \qquad \textbf{(B) } 12 \qquad \textbf{(C) } 18 \qquad \textbf{(D) } 24 \qquad \textbf{(E) } 36$

Problem 13

How many subsets of two elements can be removed from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ so that the mean (average) of the remaining numbers is 6?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 14

Which of the following integers cannot be written as the sum of four consecutive odd integers?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 72 \qquad \textbf{(D)}\ 100 \qquad \textbf{(E)}\ 200$

Problem 15

At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

$\textbf{(A)}\ 49 \qquad \textbf{(B)}\ 70 \qquad \textbf{(C)}\ 79 \qquad \textbf{(D)}\ 99 \qquad \textbf{(E)}\ 149$

Problem 16

In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

$\textbf{(A)}\ \frac{2}{15} \qquad \textbf{(B)}\ \frac{4}{11} \qquad \textbf{(C)}\ \frac{11}{30} \qquad \textbf{(D)}\ \frac{3}{8} \qquad \textbf{(E)}\ \frac{11}{15}$

Problem 17

Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 12$

Problem 18

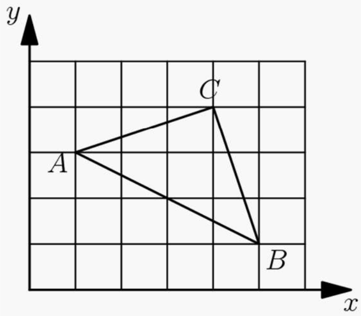
An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, $2, 5, 8, 11, 14$ is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

1				25
		X		
17				81

(A) 21 **(B)** 31 **(C)** 36 **(D)** 40 **(E)** 42

Problem 19

A triangle with vertices as $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

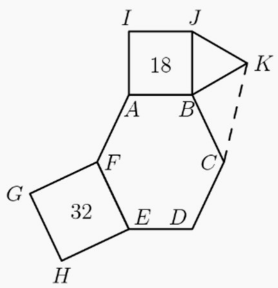
Problem 20

Ralph went to the store and bought 12 pairs of socks for a total of $\$24$. Some of the socks he bought cost $\$1$ a pair, some of the socks he bought cost $\$3$ a pair, and some of the socks he bought cost $\$4$ a pair. If he bought at least one pair of each type, how many pairs of $\$1$ socks did Ralph buy?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Problem 21

In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE = BC$. What is the area of $\triangle KBC$?



(A) $6\sqrt{2}$ **(B)** 9 **(C)** 12 **(D)** $9\sqrt{2}$ **(E)** 32

Problem 22

On June 1, a group of students are standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

$\textbf{(A) } 21 \qquad \textbf{(B) } 30 \qquad \textbf{(C) } 60 \qquad \textbf{(D) } 90 \qquad \textbf{(E) } 1080$

Problem 23

Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are $2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4,$ and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup B, then the slip with 3.5 must go into what cup?

$\textbf{(A) } A \qquad \textbf{(B) } B \qquad \textbf{(C) } C \qquad \textbf{(D) } D \qquad \textbf{(E) } E$

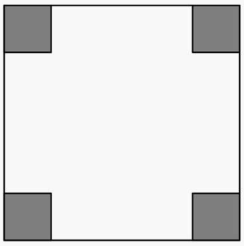
Problem 24

A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with $N > 2M$ and $M > 4$. Each team plays a 76-game schedule. How many games does a team play within its own division?

$\textbf{(A) } 36 \qquad \textbf{(B) } 48 \qquad \textbf{(C) } 54 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 72$

Problem 25

One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?



$\textbf{(A) } 9 \qquad \textbf{(B) } 12\frac{1}{2} \qquad \textbf{(C) } 15 \qquad \textbf{(D) } 15\frac{1}{2} \qquad \textbf{(E) } 17$

1. A	6. B	11. B	16. B	21. C
2. D	7. E	12. C	17. D	22. C
3. D	8. D	13. D	18. B	23. D
4. E	9. D	14. D	19. A	24. B
5. A	10. B	15. D	20. D	25. C

END No.07.

No.08 # AUSTRAL AMC - 2017

Junior Division

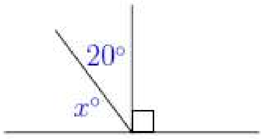
Questions 1 to 10, 3 marks each

1. The value of $2 + 0 + 1 + 7$ is

(A) 10 (B) 19 (C) 37 (D) 208 (E) 2017

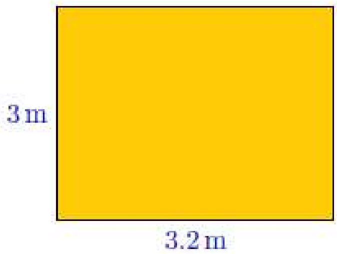
2. What is the value of x in the diagram?

(A) 20 (B) 70 (C) 80 (D) 110 (E) 160



3. This rectangle is 3.2 m wide and 3 m tall. Its area is

(A) $9.6\,\mathrm{m}^2$ (B) $15\,\mathrm{m}^2$ (C) $90.6\,\mathrm{m}^2$ (D) $9.2\,\mathrm{m}^2$ (E) $6.5\,\mathrm{m}^2$

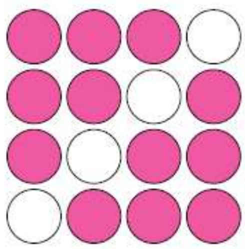


4. Starting with 13, and counting by fives, you count $13, 18, 23$, and so on. Which one of the following numbers will be one of the numbers you count?

(A) 47 (B) 48 (C) 49 (D) 50 (E) 51

5. What fraction of these circles are shaded?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{9}{16}$



6. Of the following, which digit could be put in the box to make this three-digit number divisible by 3?

$$1\,\square\,7$$

(A) 1 (B) 3 (C) 6 (D) 8 (E) 9

7. A pump runs for 150 minutes, using 8 litres of biodiesel. For how many hours can it run with 32 litres of biodiesel?

(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

---

8. Jonah returned from the shop with a bag carrying 780 g of fish, 1.35 kg of vegetables, and 680 g of fruit for his mother. The bag itself weighed 150 g. The total weight, in kilograms, that Jonah carried was

(A) 1.745 (B) 2 (C) 2.81 (D) 2.96 (E) 3

---

9. 1000% of 1 is

(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

---

10. Which one of the following numbers could be put in the box to make the fraction between 7 and 8?

$$\frac{\square + 3}{6}$$

(A) 47 (B) 25 (C) 32 (D) 37 (E) 41

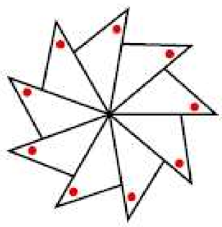
Questions 11 to 20, 4 marks each

11. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

---

12. This pinwheel star is formed by rotating a right-angled triangle around one of its corners. What is the angle at each of the nine tips that are marked with dots?



(A) 30° (B) 40° (C) 45° (D) 50° (E) 60°

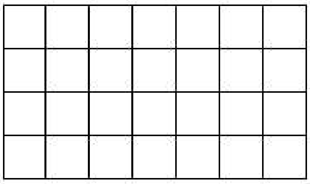
---

13. I have twelve paint tins each capable of holding twelve litres. Half of them are half full. A third of them are a third full. The rest are one-sixth full. How many litres of paint do I have in total?

(A) 48 (B) 50 (C) 52 (D) 54 (E) 56

---

14. How many ways are there of placing a single 3×1 rectangle on this grid so that it completely covers three grid squares?



(A) 34 (B) 28 (C) 56 (D) 40 (E) 10

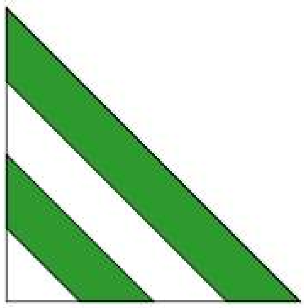
---

15. The time 2017 minutes after 10 am on Tuesday is closest to

(A) 7.30 pm Tuesday (B) 7.30 am Wednesday (C) 7.30 pm Wednesday (D) 7.30 am Thursday (E) 7.30 pm Thursday

---

16. The bottom and left side of this triangle are divided into 4 equal parts by the diagonal lines. What fraction of the large triangle is shaded?



(A) $\frac{5}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$ (E) $\frac{3}{5}$

---

17. Each of the fractions $\frac{4}{n}, \frac{5}{n}, \frac{7}{n}$ is in its simplest form. Which of the following could be the value of n?

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

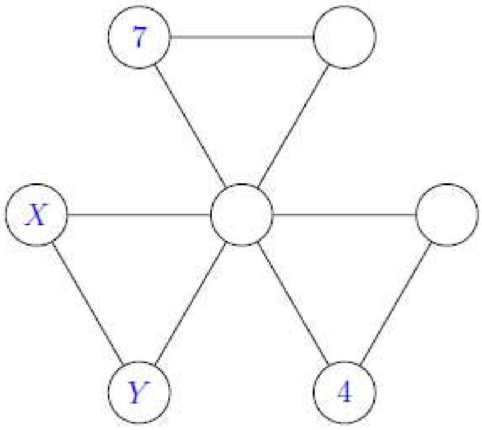
---

18. The whole numbers from 1 to 7 are to be placed in the seven circles in the diagram. In each of the three triangles drawn, the sum of the three numbers is the same.

Two of the numbers are given.

What is $X + Y$?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9



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19. Farhad, Greg and Huong were dismantling their marble madness machine and had 2017 marbles to share. They split them so that Farhad had exactly twice as many as Greg, and Greg had twice as many as Huong, with as few left over as possible. How many marbles were in Farhad's share?

(A) 1008 (B) 504 (C) 288 (D) 1344 (E) 1152

---

20. Whole numbers greater than 1 are arranged in a table in the pattern shown. In which column will the number 501 be found?

(A) A (B) B (C) C (D) D (E) E

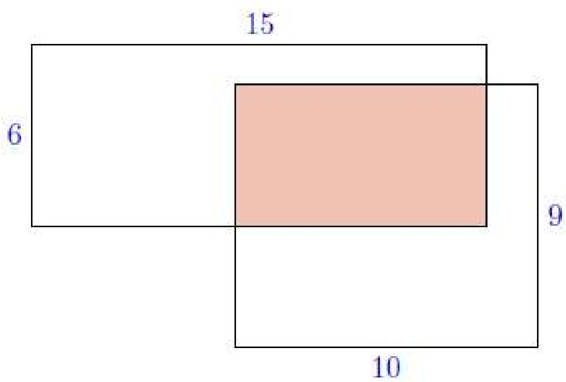
A	B	C	D	E
		2	3	4
5	6	7		
		8	9	10
11	12	13		
		14	15	16
17	18	19		
	⋮		⋮	

Questions 21 to 25, 5 marks each

21. Two rectangles overlap to create three regions, each of equal area. The original rectangles are 6 cm by 15 cm and 10 cm by 9 cm as shown. The sides of the smaller shaded rectangle are each a whole number of centimetres.

What is the perimeter of the smaller shaded rectangle, in centimetres?

(A) 24 (B) 28 (C) 30 (D) 32 (E) 36



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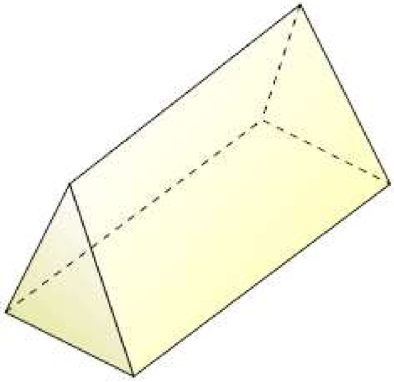
22. A number is a palindrome if it reads the same forwards as backwards. The number 131131 is a palindrome; also the first pair of digits (13), the middle pair of digits (11) and the last pair of digits (31) are prime numbers. How many such 6-digit palindromes are there?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

---

23. A triangular prism is to be cut into two pieces with a single straight cut. What is the smallest possible total for the combined number of faces of the two pieces?

(A) 6 (B) 8 (C) 9 (D) 10 (E) 11



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24. Ike and Seb were arguing over how 120 mL of soft drink had been shared between them.

To settle the argument, their dad poured one-third of Ike's drink into Seb's glass, and then he poured one-third of Seb's drink back into Ike's glass. Now they have an equal amount.

How much soft drink did Ike originally have compared to Seb?

(A) 60 mL less (B) 30 mL less (C) the same (D) 30 mL more (E) 60 mL more

---

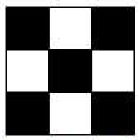
25. A 3×3 grid has a pattern of black and white squares.

A pattern is called *balanced* if each 2×2 subgrid contains exactly two squares of each colour, as seen in the first example.

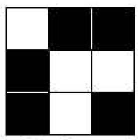
The pattern in the second example is *unbalanced* because the bottom-right 2×2 subgrid contains three white squares.

Counting rotations and reflections as different, how many balanced 3×3 patterns are there?

(A) 2 (B) 7 (C) 10 (D) 14 (E) 18



balanced



unbalanced

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

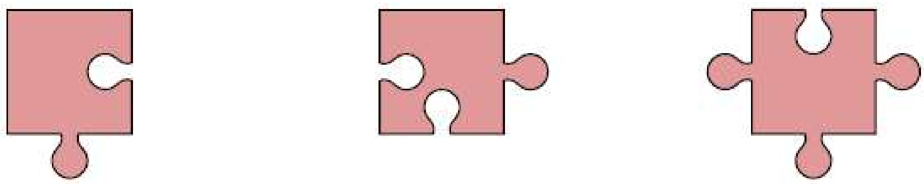
---

26. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

---

27. A jigsaw piece is formed from a square with a combination of 'tabs' and 'slots' on at least two of its sides.

Pieces are either corner, edge or interior, as shown.



corner piece (two straight sides at right angles) | edge piece (one straight side) | interior piece (no straight sides)

We treat two shapes as the same if one is a rotation of the other, without turning it over. How many different shapes are possible?

---

28. The reverse of the number 129 is 921, and these add to 1050, which is divisible by 30. How many three-digit numbers have the property that, when added to their reverse, the sum is divisible by 30?

---

29. I have a large number of toy soldiers, which I can arrange into a rectangular array consisting of a number of rows and a number of columns. I notice that if I remove 100 toy soldiers, then I can arrange the remaining ones into a rectangular array with 5 fewer rows and 5 more columns.

How many toy soldiers would I have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns?

---

30. Mike multiplied at least two consecutive integers together. He obtained a six-digit number N. The first two digits of N are 47 and the last two digits of N are 74. What is the sum of the integers that Mike multiplied together?

End. No. 08.

No 10

Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2014

Junior Section (First Round)

ultiple Choice Questions

1. Let x, y and z be real numbers satisfying $x > y > 0$ and $z \neq 0$. Which of the inequalities below is not always true?

   (A) $x+z > y+z$ (B) $x-z > y-z$ (C) $xz > yz$ (D) $\frac{1}{y}+z > \frac{1}{x}+z$
   (E) $xz^2 > yz^2$

2. If the radius of a circle is increased by 100%, the area is correspondingly increased by how many percent?

   (A) 50% (B) 100% (C) 200% (D) 300% (E) 400%

3. If $a = \sqrt{7}$, $b = \sqrt{90}$, find the value of $\sqrt{6.3}$.

   (A) $\dfrac{7b}{a\sqrt{10}}$ (B) $\dfrac{b-7a}{10}$ (C) $\dfrac{10a}{b}$ (D) $\dfrac{ab}{100}$ (E) None of the above

4. Find the value of $\dfrac{1}{1-\sqrt[4]{5}} + \dfrac{1}{1+\sqrt[4]{5}} + \dfrac{2}{1+\sqrt{5}}$.

   (A) -1 (B) 1 (C) $-\sqrt{5}$ (D) $\sqrt{5}$ (E) None of the above

5. Andrew, Catherine, Michael, Nick and Sally ordered different items for lunch. These are (in no particular order): cheese sandwich, chicken rice, duck rice, noodles and steak. Find out what Catherine had for lunch if we are given the following information:

   1. Nick sat between his friend Sally and the person who ordered steak.
   2. Michael does not like noodles.
   3. The person who ate noodles is Sally's cousin.
   4. Neither Catherine, Michael nor Nick likes rice.
   5. Andrew had duck rice.

   (A) Cheese sandwich (B) Chicken rice (C) Duck rice (D) Noodles (E) Steak

6. At 2:40 pm, the angle formed by the hour and minute hands of a clock is x°, where $0 < x < 180$. What is the value of x?

   (A) 60° (B) 80° (C) 100° (D) 120° (E) 160°

7. In the figure below, each distinct letter represents a unique digit such that the arithmetic sum holds. If the letter L represents 9, what is the digit represented by the letter T?

$$\begin{array}{ccccccccc} & \mathrm{T} & \mathrm{E} & \mathrm{R} & \mathrm{R} & \mathrm{I} & \mathrm{B} & \mathrm{L} & \mathrm{E} \\ + & & & \mathrm{N} & \mathrm{U} & \mathrm{M} & \mathrm{B} & \mathrm{E} & \mathrm{R} \\ \hline & \mathrm{T} & \mathrm{H} & \mathrm{I} & \mathrm{R} & \mathrm{T} & \mathrm{E} & \mathrm{E} & \mathrm{N}. \end{array}$$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

8. A regular cube is to have 2 faces coloured red, 2 faces coloured blue and 2 faces coloured orange. We consider two colourings to be the same if one can be obtained by a rotation of the cube from another. How many different colourings are there?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 9

9. In $\triangle ABC$, $AB = AC$, $\angle BAC = 120°$, D is the midpoint of BC, and E is a point on AB such that DE is perpendicular to AB. Find the ratio $AE : BD$.

(A) $1:2$ (B) $2:3$ (C) $1:\sqrt{3}$ (D) $1:2\sqrt{3}$ (E) $2:3\sqrt{3}$

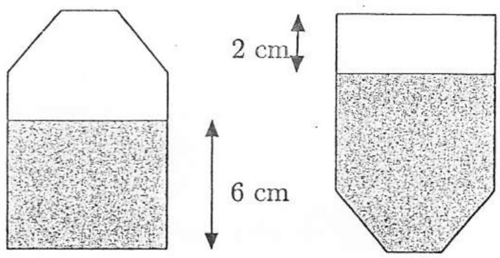
10. How many ways are there to add four positive odd numbers to get a sum of 22?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

Short Questions

11. Successive discounts of 10% and 20% are equivalent to a single discount of x%. What is the value of x?

12. The diagram below shows the front view of a container with a rectangular base. The container is filled with water up to a height of 6 cm. If the container is turned upside down, the height of the empty space is 2 cm. Given that the total volume of the container is 28 cm^3, find the volume of the water in cm^3.



13. Let A be the solution of the equation

$$\frac{x-7}{x-8} - \frac{x-8}{x-9} = \frac{x-10}{x-11} - \frac{x-11}{x-12}.$$

Find the value of $6A$.

13. Let A be the solution of the equation

$$\frac{x-7}{x-8} - \frac{x-8}{x-9} = \frac{x-10}{x-11} - \frac{x-11}{x-12}.$$

Find the value of $6A$.

14. The sum of the two smallest positive divisors of an integer N is 6, while the sum of the two largest positive divisors of N is 1122. Find N.

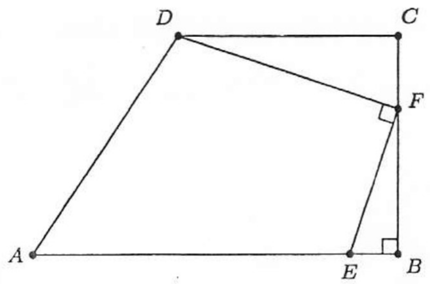
15. Let D be the absolute value of the difference of the two roots of the equation $3x^2 - 10x - 201 = 0$. Find $\lfloor D \rfloor$.

16. If m and n are positive real numbers satisfying the equation

$$m + 4\sqrt{mn} - 2\sqrt{m} - 4\sqrt{n} + 4n = 3,$$

find the value of $\dfrac{\sqrt{m} + 2\sqrt{n} + 2014}{4 - \sqrt{m} - 2\sqrt{n}}$.

17. In the diagram below, $ABCD$ is a trapezium with $AB \parallel DC$ and $\angle ABC = 90°$. Points E and F lie on AB and BC respectively such that $\angle EFD = 90°$. If $CD + DF = BC = 4$, find the perimeter of $\triangle BFE$.



18. If p, q and r are prime numbers such that their product is 19 times their sum, find $p^2 + q^2 + r^2$.

19. John received a box containing some marbles. Upon inspecting the marbles, he immediately discarded 7 that were chipped. He then gave one-fifth of the marbles to his brother. After adding the remaining marbles to his original collection of 14, John discovered that he could divide his marbles into groups of 6 with exactly 2 left over or he could divide his marbles into groups of 5 with none left over. What is the smallest possible number of marbles that John received from the box?

20. Let N be a 4-digit number with the property that when all the digits of N are added to N itself, the total equals 2019. Find the sum of all the possible values of N.

21. There are exactly two ways to insert the numbers $1, 2$ and 3 into the circles

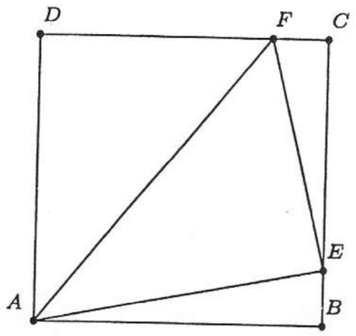
$$\bigcirc < \bigcirc > \bigcirc$$

such that every order relation $<$ or $>$ between numbers in adjacent circles is satisfied. The two ways are ①< ③> ② and ②< ③> ①.

Find the total number of possible ways to insert the numbers $3, 14, 15, 9, 2$ and 6 into the circles below, such that every order relation $<$ or $>$ between the numbers in adjacent pairs of circles is satisfied.

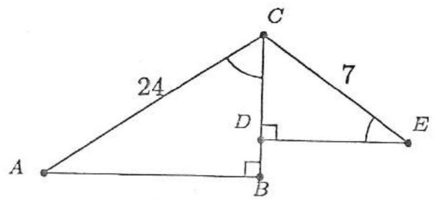
$$\bigcirc > \bigcirc > \bigcirc > \bigcirc < \bigcirc < \bigcirc .$$

22. Let $ABCD$ be a square of sides 8 cm. If E and F are variable points on BC and CD respectively such that $BE = CF$, find the smallest possible area of the triangle $\triangle AEF$ in cm^2.



23. If a, b and c are non-zero real numbers satisfying $a+2b+3c = 2014$ and $2a+3b+2c = 2014$, find the value of $\dfrac{a^2+b^2+c^2}{ac+bc-ab}$.

24. In the diagram below, $\triangle ABC$ and $\triangle CDE$ are two right-angled triangles with $AC = 24$, $CE = 7$ and $\angle ACB = \angle CED$. Find the length of the line segment AE.



25. The hypotenuse of a right-angled triangle is 10 and the radius of the inscribed circle is 1. Find the perimeter of the triangle.

26. Let x be a real number satisfying $\left(x+\frac{1}{x}\right)^2 = 3$. Evaluate $x^3 + \frac{1}{x^3}$.

27. For $2 \le x \le 8$, we define $f(x) = |x-2| + |x-4| - |2x-6|$. Find the sum of the largest and smallest values of $f(x)$.

28. If both n and $\sqrt{n^2 + 204n}$ are positive integers, find the maximum value of n.

29. Let $N = \overline{abcd}$ be a 4-digit perfect square that satisfies $\overline{ab} = 3 \cdot \overline{cd} + 1$. Find the sum of all possible values of N.

(The notation $n = \overline{ab}$ means that n is a 2-digit number and its value is given by $n = 10a+b$.)

30. Find the following sum:

$$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots+\frac{1}{29}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots+\frac{2}{29}\right)$$
$$+\left(\frac{3}{4}+\frac{3}{5}++\cdots+\frac{3}{29}\right)+\cdots+\left(\frac{27}{28}+\frac{27}{29}\right)+\frac{28}{29}.$$

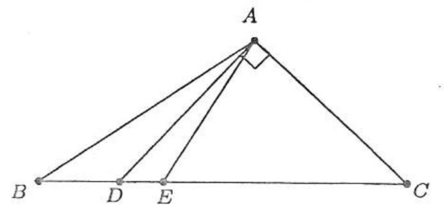
31. If $ax + by = 7$, $ax^2 + by^2 = 49$, $ax^3 + by^3 = 133$, and $ax^4 + by^4 = 406$, find the value of $2014(x + y - xy) - 100(a + b)$.

32. For $a \ge \frac{1}{8}$, we define

$$g(a) = \sqrt[3]{a + \frac{a+1}{3}\sqrt{\frac{8a-1}{3}}} + \sqrt[3]{a - \frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}.$$

Find the maximum value of $g(a)$.

33. In the diagram below, AD is perpendicular to AC and $\angle BAD = \angle DAE = 12°$. If $AB + AE = BC$, find $\angle ABC$.



34. Define S to be the set consisting of positive integers n, such that the inequalities

$$\frac{9}{17} < \frac{n}{n+k} < \frac{8}{15},$$

hold for *exactly one* positive integer k. Find the largest element of S.

35. The number 2^{29} has exactly 9 distinct digits. Which digit is missing?

SMO 2014 (Junior Section) Answers

1. C	11. 28	21. 10	31. 5956
2. D	12. 21	22. 24	32. 1
3. E	13. 60	23. 2	33. 44
4. A	14. 935	24. 25	34. 144
5. D	15. 16	25. 22	35. 4
6. E	16. 2017	26. 0 (question is wrong as x is not real!)	
7. A	17. 8	27. 2	
8. C	18. 491	28. 2500	
9. D	19. 52	29. 2809	
10. E	20. 4008	30. 203	

End No. 09.

No. 10

Hanoi Open Mathematical Competition 2016

Junior Section

Question 1. If

$$2016 = 2^5 + 2^6 + \cdots + 2^m,$$

then m is equal to

(A): 8 (B): 9 (C): 10 (D): 11 (E): None of the above.

Question 2. The number of all positive integers n such that

$$n + s(n) = 2016,$$

where $s(n)$ is the sum of all digits of n, is

(A): 1 (B): 2 (C): 3 (D): 4 (E): None of the above.

Question 3. Given two positive numbers a, b such that $a^3 + b^3 = a^5 + b^5$, then the greatest value of $M = a^2 + b^2 - ab$ is

(A): $\frac{1}{4}$ (B): $\frac{1}{2}$ (C): 2 (D): 1 (E): None of the above.

Question 4. A monkey in Zoo becomes lucky if he eats three different fruits. What is the largest number of monkeys one can make lucky, by having 20 oranges, 30 bananas, 40 peaches and 50 tangerines? Justify your answer.

(A): 30 (B): 35 (C): 40 (D): 45 (E): None of the above.

Question 5. There are positive integers x, y such that $3x^2 + x = 4y^2 + y$, and $(x - y)$ is equal to

(A): 2013 (B): 2014 (C): 2015 (D): 2016 (E): None of the above.

Question 6. Determine the smallest positive number a such that the number of all integers belonging to $(a, 2016a]$ is 2016.

Question 7. Nine points form a grid of size 3×3. How many triangles are there with 3 vetices at these points?

Question 8. Find all positive integers x, y, z such that

$$x^3 - (x + y + z)^2 = (y + z)^3 + 34.$$

Question 9. Let x, y, z satisfy the following inequalities

$$\begin{cases} |x+2y-3z| \le 6 \\ |x-2y+3z| \le 6 \\ |x-2y-3z| \le 6 \\ |x+2y+3z| \le 6 \end{cases}$$

Determine the greatest value of $M = |x| + |y| + |z|$.

Question 10. Let h_a, h_b, h_c and r be the lengths of altitudes and radius of the inscribed circle of ΔABC, respectively. Prove that

$$h_a + 4h_b + 9h_c > 36r.$$

Question 11. Let be given a triangle ABC, and let I be the middle point of BC. The straight line d passing I intersects AB, AC at M, N, respectively. The straight line d' ($\not\equiv d$) passing I intersects AB, AC at Q, P, respectively. Suppose M, P are on the same side of BC and MP, NQ intersect BC at E and F, respectively. Prove that $IE = IF$.

Question 12. In the trapezoid $ABCD$, $AB \parallel CD$ and the diagonals intersect at O. The points P, Q are on AD, BC respectively such that $\angle APB = \angle CPD$ and $\angle AQB = \angle CQD$. Show that $OP = OQ$.

Question 13. Let H be orthocenter of the triangle ABC. Let d_1, d_2 be lines perpendicular to each-another at H. The line d_1 intersects AB, AC at D, E and the line d_2 intersects BC at F. Prove that H is the midpoint of segment DE if and only if F is the midpoint of segment BC.

Question 14. Given natural numbers a, b such that $2015a^2 + a = 2016b^2 + b$. Prove that $\sqrt{a-b}$ is a natural number.

Question 15. Find all polynomials of degree 3 with integer coefficients such that $f(2014) = 2015$, $f(2015) = 2016$, and $f(2013) - f(2016)$ is a prime number.

Hints and Solutions

Question 1. (C).

Question 2. (B): $n = 1989, 2007$.

Question 3. (D).

We have

$$ab(a^2-b^2)^2 \ge 0 \Leftrightarrow 2a^3b^3 \le ab^5 + a^5b \Leftrightarrow (a^3+b^3)^2 \le (a+b)(a^5+b^5). \qquad (1)$$

Combining $a^3+b^3=a^5+b^5$ and (1), we find

$$a^3+b^3 \leq a+b \Leftrightarrow a^2+b^2-ab \leq 1.$$

The equality holds if $a=1, b=1$.

Question 4. (D).

First we leave tangerines on the side. We have $20+30+40=90$ fruites. As we feed the happy monkey is not more than one tangerine, each monkey eats fruits of these 90 at least 2.

Hence, the monkeys are not more than $90/2=45$. We will show how you can bring happiness to 45 monkeys:

5 monkeys eat: orange, banana, tangerine;

15 monkeys eat: orange, peach, tangerine;

25 Monkeys eat peach, banana, tangerine.

At all 45 lucky monkeys - and left five unused tangerines!

Question 5. (E). Since $x-y$ is a square.

We have $3x^2+x=4y^2+y \Leftrightarrow (x-y)(3x+3y+1)=y^2$.

We prove that $(x-y; 3x+3y+1)=1$.

Indeed, if $d=(x-y; 3x+3y+1)$ then y^2 is divisible by d^2 and y is divisible by d; x is divisible by d, i.e. 1 is divisible by d, i.e. $d=1$.

Since $x-y$ and $3x+3y+1$ are prime relative then $x-y$ is a perfect square.

Question 6. The smallest integer greater than a is $[a]+1$ and the largest integer less than or is equal to $2016a$ is $[2016a]$. Hence, the number of all integers belonging to $(a, 2016a]$ is $[2016a]-[a]$.

Now we difine the smallest positive number a such that

$$[2016a]-[a]=2016.$$

If $0<a\leq 1$ then $[2016a]-[a]<2016$.

If $a\geq 2$ then $[2016a]-[a]>2016$.

Let $a=1+b$, where $0<b<1$. Then $[a]=1$, $[2016a]=2016+[2016b]$ and $[2016a]-[a]=2015+[2016b]=2016$ iff $[2016b]=1$. Hence the smallest positive number b such that $[2016b]=1$ is $b=\dfrac{1}{2016}$.

Thus, $a=1+\dfrac{1}{2016}$ is a smallest positive number such that the number of all integers belonging to $(a, 2016a]$ is 2016.

Question 7. We divide the triangles into two types:

Type 1: Two vertices lie in one horizontal line, the third vertice lies in another horizontal lines.

For this type we have 3 possibilities to choose the first line, 2 posibilities to choose 2nd line. In first line we have 3 possibilities to choose 2 vertices, in the second line we have 3 possibilities to choose 1 vertex. In total we have $3 \times 2 \times 3 \times 3 = 54$ triangles of first type.

Type 2: Three vertices lie in distinct horizontal lines.

We have $3 \times 3 \times 3$ triangles of these type. But we should remove degenerated triangles from them. There are 5 of those (3 vertical lines and two diagonals). So, we have 27 - 5 = 22 triangles of this type.

Total we have 54 + 22 = 76 triangles.

For those students who know about C_n^k this problem can be also solved as $C_9^3 - 8$ where 8 is the number of degenerated triangles.

Question 8. Putting $y + z = a$, $a \in \mathbb{Z}$, $a \geq 2$, we have

$$x^3 - a^3 = (x+a)^2 + 34. \tag{1}$$

$$\Leftrightarrow (x-a)\left(x^2 + xa + a^2\right) = x^2 + 2ax + a^2 + 34. \tag{2}$$

$$\Leftrightarrow (x-a-1)\left(x^2 + xa + a^2\right) = xa + 34.$$

Since x, a are integers, we have $x^2 + xa + a^2 \geq 0$ and $xa + 34 > 0$. That follow $x - a - 1 > 0$, i.e. $x - a \geq 2$.

This and (2) together imply

$$x^2 + 2ax + a^2 + 34 \geq 2\left(x^2 + xa + a^2\right) \Leftrightarrow x^2 + a^2 \leq 34.$$

Hence $x^2 < 34$ and $x < 6$.

On the other hand, $x \geq a + 2 \geq 4$ then $x \in \{4, 5\}$.

If $x = 5$, then from $x^2 + a^2 \leq 34$ it follows $2 \leq a \leq 3$. Thus $a \in \{2, 3\}$.

The case of $x = 5$, $a = 2$ does not satisfy (1) for $x = 5$, $a = 3$, from (1) we find $y = 1, z = 2$ or $y = 2, z = 1$,

If $x = 4$, then from the inequality $x - a \geq 2$ we find $a \leq 2$, which contradicts to (1).

Conclusion: $(x, y, z) = (5, 1, 2)$ and $(x, y, z) = (5, 2, 1)$.

Question 9. Note that for all real numbers a, b, c, we have

$$|a| + |b| = \max\{|a+b|, |a-b|\}$$

and

$$|a| + |b| + |c| = \max\{|a+b+c|, |a+b-c|, |a-b-c|, |a-b+c|\}.$$

Hence

$$M = |x| + |y| + |z| \leq |x| + 2|y| + 3|z| = |x| + |2y| + |3z|$$
$$= \max\{|x+y+z|, |x+y-z|, |x-y-z|, |x-y+z|\} \leq 6.$$

Thus $\max M = 6$ when $x = \pm 6$, $y = z = 0$.

Question 10. Let a, b, c be the side-lengths of ΔABC corresponding to h_a, h_b, h_c and S be the area of ΔABC. Then

$$ah_a = bh_b = ch_c = (a+b+c) \times r = 2S.$$

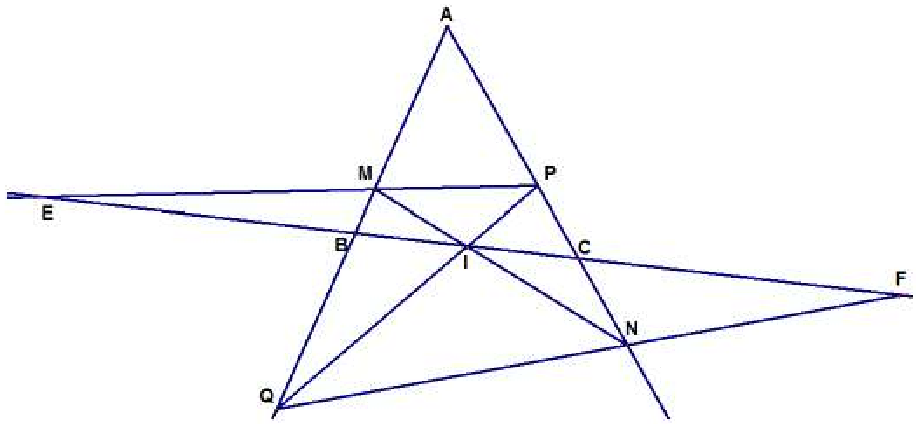
Hence

$$h_a + 4h_b + 9h_c = \frac{2S}{a} = \frac{8S}{b} = \frac{18S}{c}$$

$$= 2S\left(\frac{1^2}{a} + \frac{2^2}{b} + \frac{3^2}{c}\right) \geq 2S\frac{(1+2+3)^2}{a+b+c} = (a+b+c)\,r\frac{(1+2+3)^2}{a+b+c} = 36r.$$

The equality holds iff $a : b : c = 1 : 2 : 3$ (it is not posible for $a + b > c$).

Question 11. Since $IB = IC$ then it is enough to show $\dfrac{EB}{EC} = \dfrac{FC}{FB}$.



By Menelaus theorem:

- For ΔABC and three points E, M, P, we have

$$\frac{EB}{EC} \times \frac{PC}{PA} \times \frac{MA}{MB} = 1$$

then

$$\frac{EB}{EC} = \frac{PA}{PC} \times \frac{MB}{MA}. \tag{1}$$

- For ΔABC and three points F, N, Q, we have

$$\frac{FC}{FB} \times \frac{QB}{QA} \times \frac{NA}{NC} = 1$$

then

$$\frac{FC}{FB} = \frac{NC}{NA} \times \frac{QA}{QB}. \tag{2}$$

- For ΔABC and three points M, I, N, we have

$$\frac{MB}{MA} \times \frac{NA}{NC} \times \frac{IC}{IB} = 1.$$

Compare with $IB = IC$ we find

$$\frac{MB}{MA} = \frac{NC}{NA}. \tag{3}$$

- For ΔABC and three points Q, I, P, we have

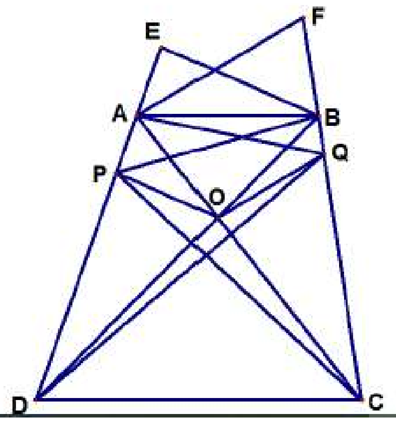
$$\frac{PA}{PC} \times \frac{IC}{IB} \times \frac{QB}{QA} = 1$$

then

$$\frac{PA}{PC} = \frac{QA}{QB}. \tag{4}$$

Equalities (1), (2), (3) and (4) toghether imply $IE = IF$.

Question 12.



Extending DA to B' such that $BB' = BA$, we find $\angle PB'B = \angle B'AB = \angle PDC$ and then triangles DPC and $B'PB$ are similar.

It follows that $\dfrac{DP}{PB'} = \dfrac{CD}{BB'} = \dfrac{CD}{BA} = \dfrac{DO}{BO}$ and so $PO \parallel BB'$.

Since triangles DPO and $DB'B$ are similar, we have

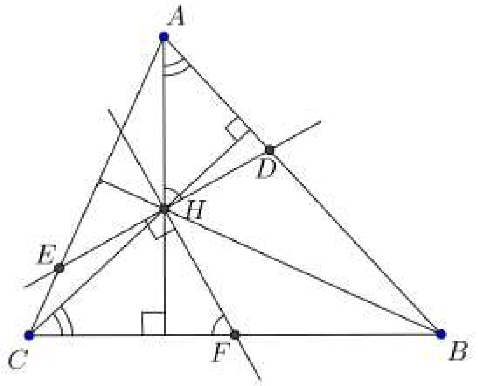
$$OP = BB' \times \frac{DO}{DB} = AB \times \frac{DO}{DB}.$$

Similarly, we have $OQ = AB \times \dfrac{CO}{CA}$ and it follows $OP = OQ$.

Question 13. Since $HD \perp HF, HA \perp FC$ and $HC \perp DA, \angle DAH = \angle HCF$ and $\angle DHA = \angle HFC$, therefore the triangles DHA, HFC are similar.

So $$\frac{HA}{HD} = \frac{FC}{FH} \tag{1}$$

Similarly, $\triangle EHA \backsim \triangle HFB$, so $$\frac{HE}{HA} = \frac{FH}{FB} \tag{2}$$



From (1) and (2), obtained $\dfrac{HE}{HD} = \dfrac{FC}{FB}$.
It follows H is midpoint of the segment DE iff F is midpoint of the segment BC.

Question 14. From equality

$$2015a^2 + a = 2016b^2 + b, \tag{1}$$

we find $a \geq b$.

If $a = b$ then from (1) we have $a = b = 0$ and $\sqrt{a-b} = 0$.

If $a > b$, we write (1) as

$$b^2 = 2015(a^2 - b^2) + (a - b) \Leftrightarrow b^2 = (a - b)(2015a + 2015b + 1). \tag{2}$$

Let $(a, b) = d$ then $a = md$, $b = nd$, where $(m, n) = 1$. Since $a > b$ then $m > n$, and put $m - n = t > 0$.

Let $(t, n) = u$ then n is divisible by u, t is divisible by u and m is divisible by u. That follows $u = 1$ and then $(t, n) = 1$.

Putting $b = nd$, $a - b = td$ in (2), we find

$$n^2 d = t(2015dt + 4030dn + 1). \tag{3}$$

From (3) we get $n^2 d$ is divisible by t and compaire with $(t, n) = 1$, it follows d is divisible by t.

Also from (3) we get $n^2 d = 2015dt^2 + 4030dnt + t$ and then $t = n^2 d - 2015dt^2 - 4030dnt$.

Hence $t = d(n^2 - 2015t^2 - 4030nt)$, i.e. t is divisible by d, i.e. $t = d$ and then $a - b = td = d^2$ and $\sqrt{a-b} = d$ is a natural number.

Question 15. Let $g(x) = f(x) - x - 1$. Then $g(2014) = f(2014) - 2014 - 1 = 0$, $g(2015) = 2016 - 2015 - 1 = 0$. Hence $g(x) = (ax + b)(x - 2014)(x - 2015)$ and

$$f(x) = (ax + b)(x - 2014)(x - 2015) + x + 1, \;\; a, b \in \mathbb{Z}, a \neq 0.$$

We have $f(2013) = 2(2013a + b) + 2014$ and

$$f(2016) = 2(2016a + b) + 2017.$$

That follows

$$f(2013)-f(2016) = 2(2013a+b)+2014-[2(2016a+b)+2017] = -6a-3 = 3(-2a-1)$$

and $f(2013) - f(2016)$ is prime iff $-2a - 1 = 1$, i.e. $a = -1$.

Conlusion: All polynomials of degree 3 with integer coefficients such that $f(2014) = 2015$, $f(2015) = 2016$ and $f(2013) - f(2016)$ is a prime number are of the form

$$f(x) = (b - x)(x - 2014)(x - 2015) + x + 1, \quad b \in \mathbb{Z}.$$

End No.10.