6D-Math

BÀI TOÁN HAY - LỜI GIẢI ĐẸP

VOL 1

108 BÀI TOÁN CHỌN LỌC

Hà Nội - 2016

LỜI NÓI ĐẦU

Quyển sách này gồm 108 bài toán được chọn lọc từ những đề tài đã và đang được học sinh, các thầy giáo, các cô giáo, và các bạn yêu toán quan tâm. Đó là các bài toán trong hình học, đại số, tổ hợp, số học và logic. Chúng tôi hy vọng sẽ mang đến bạn đọc những bài toán trong sáng, gần gũi, thân thiện và tạo nhiều cảm hứng.

Chúng tôi cho rằng, một chương trình bồi dưỡng và phát triển tài năng Toán học nên được xây dựng bằng công nghệ giáo dục khác biệt, đáp ứng tiêu chí giáo dục tiếp cận năng lực, thay vì giáo dục tiếp cận kiến thức. Với một chương trình tích hợp được xây dựng một cách thống nhất cùng với đội ngũ giảng dạy biết cách truyền tải và hoạt động theo nhóm, luôn đề cao vai trò của sự tương tác giữa học sinh và giáo viên, học sinh và học sinh, giáo viên và giáo viên. Mong rằng cuốn sách nhỏ này sẽ là một sự khởi đầu của các cuốn sách tiếp theo của chúng tôi về những **Bài toán hay-Lời giải đẹp**, và hơn thế nữa...!
Ban biên tập chân thành cảm ơn những đóng góp xây dựng của bạn đọc, để những tài liệu tiếp theo của chúng tôi sẽ được hoàn chỉnh hơn.

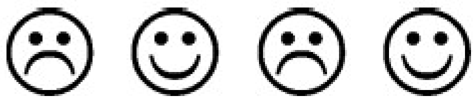
Hà nội, tháng 9 năm 2016

Nhóm 6D-Math

Sách chỉ để tặng

PROBLEMS

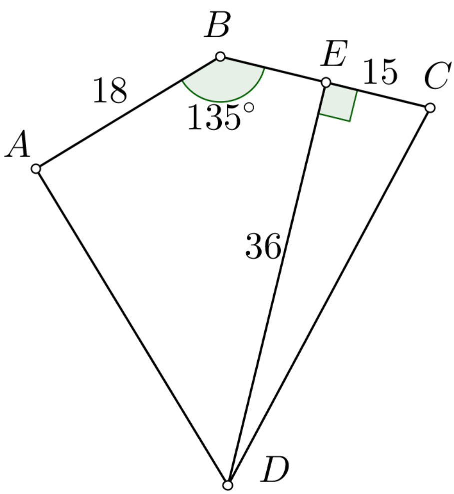
1. There are four buttons in a row, as shown below. Two of them show happy faces, and two of them show sad faces. If we press on a face, its expression turns to the opposite (e.g. a happy face turns into a sad face). In addition, the adjacent buttons also change their expressions. What is the least number of times you need to press a button in order to turn them all into happy faces?



2. Place the numbers 1 to 9 in the empty white boxes so that the 3 horizontal and 3 vertical equations are true. Each digit can be used exactly once. Calculations are done from left to right and from top to bottom.

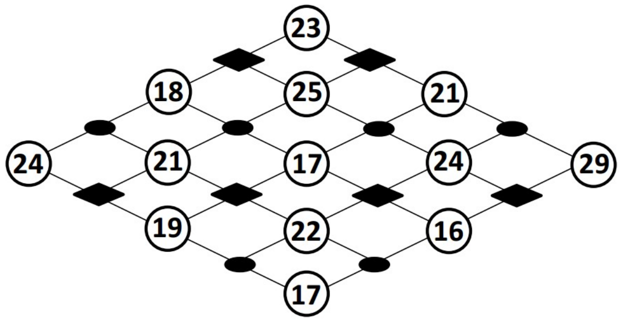
	+		÷		=	9
+		÷		+		
	×		+		=	26
-		+		-		
	+		+		=	16
=		=		=		
8		10		4		

3. $ABCD$ is a quadrilateral $\angle BAD = \angle CED = 90°$, $\angle ABC = 135°$, $AB = 18$cm, $CE = 15$cm, $DE = 36$cm. Find the area of the quadrilateral $ABCD$.

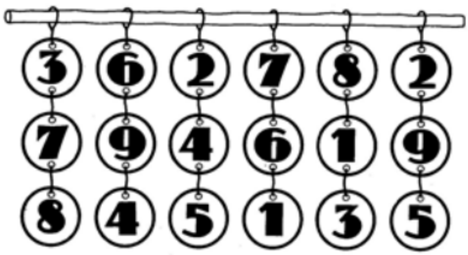


4. Starting from the far left circle, move along the lines to the far right circle, collect the numbers in the

circles, the diamonds and the ovals as you go (each can be picked only once). The ovals equal -10 and the diamonds equal -15, respectively. What are the minimum and maximum total sums you can gain?



5. Each number from one to nine appears twice on the eighteen disks that are hanging by threads. Your task is to cut the least number of threads to leave only nine disks hanging that have each number from one to nine. Find the least number of threads you need to cut.

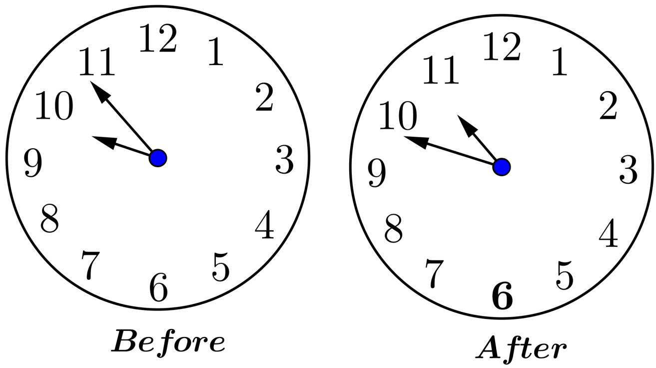


6. Let $S = \dfrac{2}{1 \times 3} + \dfrac{2^2}{3 \times 5} + \cdots + \dfrac{2^{49}}{97 \times 99}$ and $T = \dfrac{1}{3} + \dfrac{2}{5} + \dfrac{2^2}{7} + \cdots + \dfrac{2^{48}}{99}$, then find the value of $S - T$.

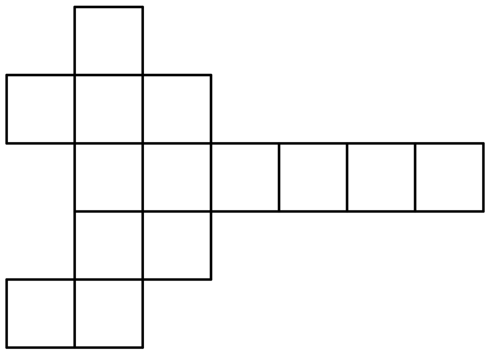
7. Two smart students A and B participate in a mental quiz bowl. The Quizmaster reads the question, "Guess a two-digit number that can be divided by 7. I have two cube cards, each with a number printed on them. The number on the first card represents the sum of the digits of this number, while the product of the number's two digits is printed on the second card. Each of you will pick one card and do the analysis on your own". After reading the card, each of them say that they cannot predict what the two-digit number is, but right after listening to each other's statement, they immediately say, "I know", and they both give the correct answer. What is the number?

8. On Saturday, Jimmy started painting his toy helicopter between 9:00 am and 10:00 am. When he finished between 10:00 and 11:00 am on the same morning, he found the hour and minute hands exactly switched places: the hour hand was exactly where the minute hand had been, and the minute

hand was exactly where the hour hand had been when he started. Jimmy spent t hours painting. Determine the value of t.

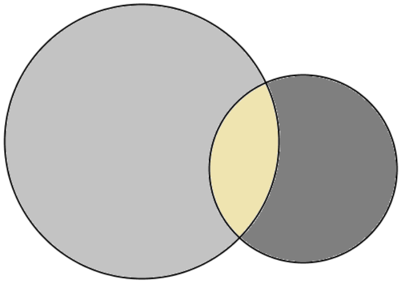


9. Hoa likes to build models of three dimensional objects from square ruled paper. Last time she used scissors to cut out a shape as shown in the figure below. Then she glued it together in such a way that no two squares were overlapping, there were no holes on the surface of the resultant object and it had nonzero volume. How many vertices did this object have? Note, that by a vertex we mean a vertex of the three-dimensional object, not a lattice point on the paper.



10. 32 teams are competing in a basketball tournament. At each stage, the teams are divided into groups of 4. In each group, every team plays exactly once against every other team. The best two teams are qualified for the next round, while the other two are eliminated. After the last stage, the two remaining teams play one final match to determine the winner. How many matches will be played in the whole tournament?

11. By drawing two circles, Mike obtained a figure, which consists of three regions (see picture). What is the largest number of regions he could obtain by drawing two squares?



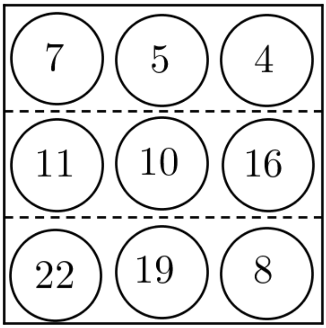
12. During the final game of a soccer championship the teams scored a lot of goals. Six goals were scored during the first period of the game and the guest team was leading at the halftime break. During the second period, the home team scored 3 goals and, as a result, they won the game. How many goals did the home team score altogether?

13. Twenty girls stood in a row, facing right. Four boys joined the row, but facing left. Each boy counted the number of girls in front of him. The numbers were 3, 6, 15 and 18, respectively. Each girl also counted the number of boys in front of her. What was the sum of the numbers counted by the girls?

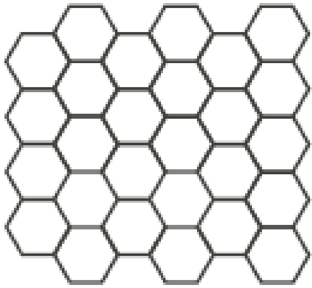
14. Six boys share an apartment with two bathrooms, which they use every morning beginning at 7:00 am. There is never more than one person in either bathroom at any one time. They spend 8, 10, 12, 17, 21

and 22 minutes at a stretch in the bathroom, respectively. What is the earliest time they can finish using the bathrooms?

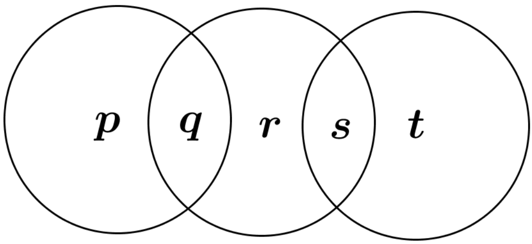
15. A puzzle starts with nine numbers placed in a grid, as shown below. At each move, you are allowed to swap any two numbers. The aim is to arrange the numbers in a way that the sum total of each row is a multiple of 3. What is the smallest number of moves needed?



16. Serena colours the hexagons on the tiling shown below. If two hexagons share a side, she colours them with different coloured pencils. What is the least number of colours that she can use to colour all of the hexagons?



17. In the diagram, p, q, r, s, and t represent five consecutive integers, not necessarily in order. The sum of the two integers in the leftmost circle is 63. The two integers in the rightmost circle add up to 57. What is the value of r?



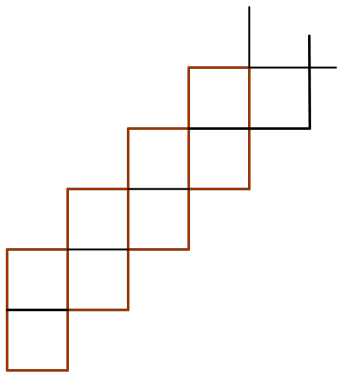
18. In the next line insert "+" signs between the numbers as many times as you want so that the result is a correct equality. $987654321 = 90$. Example: $9 + 8 + 7 + 65 + 4 + 3 + 21 = 117$.

19. Somebody placed the digits $1, 2, 3, \ldots, 9$ around the circumference of a circle in an arbitrary order. By

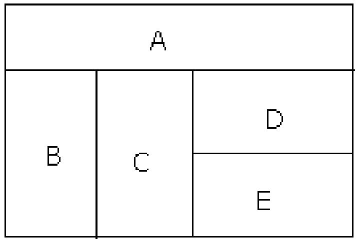
reading three consecutive digits clockwise, you get a 3-digit whole number. There are nine such 3-digit numbers altogether. Find their sum.

20. Given are two three-digit numbers a and b and a four-digit number c. If the sums of the digits of the numbers $a+b$, $b+c$ and $c+a$ are all equal to 3, find the largest possible sum of the number $a+b+c$.

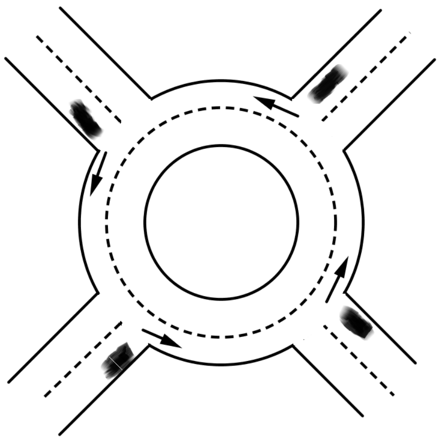
21. A shape consisting of 2016 small squares is made by continuing the pattern shown in the diagram. The small squares have sides of 1 cm each. What is the length, in cm, of the perimeter of the whole shape?



22. In how many ways can each region of the figure be coloured using 4 different colours so that no adjacent ones will have the same colour?

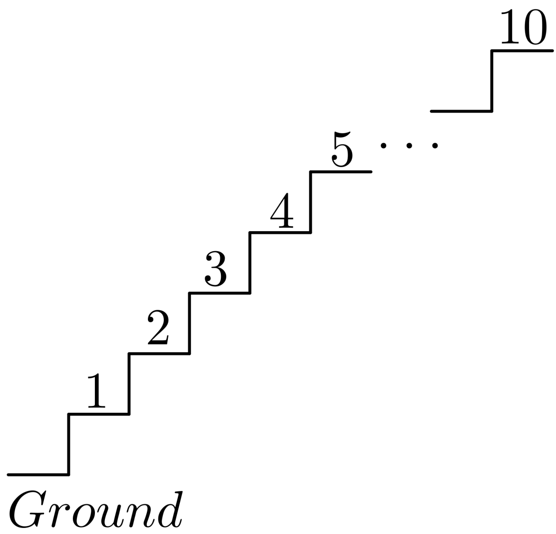


23. Four cars enter a roundabout at the same time, each one from a different direction, as shown in the diagram. Each of the cars drives less than a full round, and no two cars leave the roundabout at the same exit. How many different ways are there for the cars to leave the roundabout?

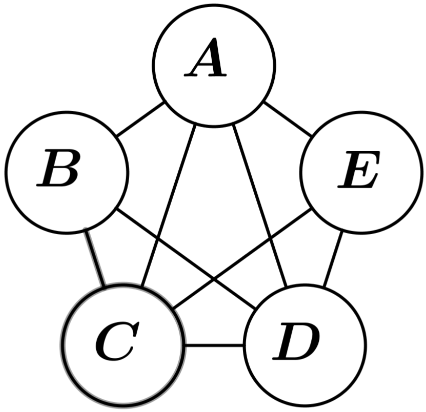


24. A staircase has 10 steps. If Peter can climb either

1 or 2 steps each time, in how many ways can he reach the top?



25. The figure shows five circles A, B, C, D and E. They are to be painted, each in one colour. Two circles joined by a line segment must have different colours. If five colours are available, how many different ways of painting are there?



26. Find the sum of the number pattern below:

$$\begin{array}{lllll} 1 & 2 & 3 & \dots & 30 \\ 2 & 3 & 4 & \dots & 31 \\ 3 & 4 & 5 & \dots & 32 \\ \dots & & & & \\ 30 & 31 & 32 & \dots & 59. \end{array}$$

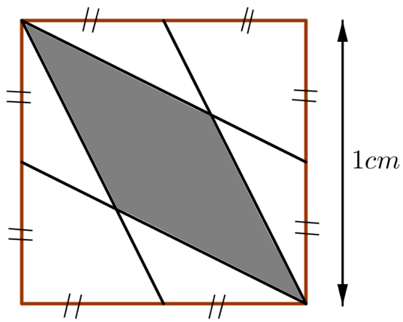
27. Five kids A, B, C, D and E are sitting around a circular table with some candies. Each of them gets an even number of candies. The quantities are $10, 30, 20$, 20 and 40, respectively. In the first round, each of them gives one half of their candies to the kid to their right. At this time, the amounts of their candies become $25, 20, 25, 20$ and 30, respectively. If the kid's number of candies is odd, then he she should pick one from the table. Is it possible that the

kids have the same number of candies after several rounds? How many pieces would everyone have? If it is possible, please write down the process. Explain your reasoning if it is not possible.

28. Integer numbers are filled in a square grid in a pattern shown below. Which column and which row contain number 2000?

1	**2**	**9**	**10**	**25**		
4	**3**	**8**	**11**	**24**		
5	**6**	**7**	**12**	**23**		
16	**15**	**14**	**13**	**22**		
17	**18**	**19**	**20**	**21**		

29. Find the area of the shaded part in below figure:



30. Money in Wonderland comes in \$5 and \$7 bills.

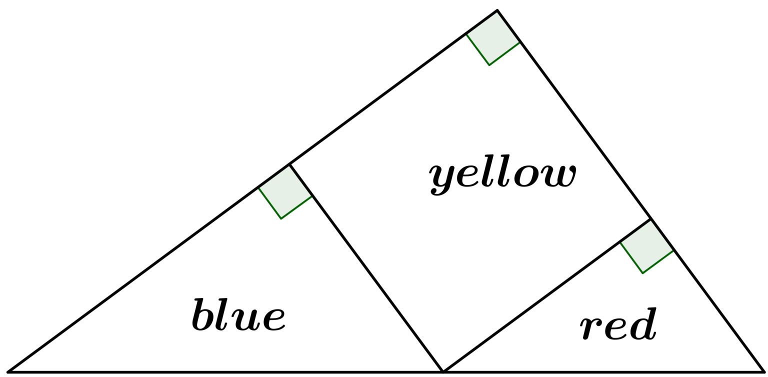
   (a) What is the smallest amount of money you need to have in order to buy a slice of pizza which costs \$1 and get your change in full? (The pizza man has plenty of \$5 and \$7 bills.) For example, having \$7 won't do, since the pizza man can only give you \$7 back.

   (b) Vending machines in Wonderland accept only exact payments (do not give back change). List all positive integer numbers which CANNOT be used as prices in such vending machines. (That is, find the sums of money that cannot be paid by exact change.)

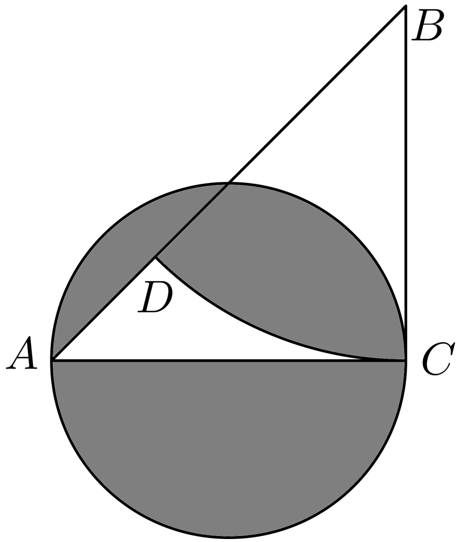
31. The "4" button on my calculator is defective, so I cannot enter numbers which contain the digit 4. Moreover, my calculator does not display the digit 4 if it is part of an answer. Thus, I cannot enter the calculation 2×14 and do not attempt to do so. Also, the result of multiplying 3 by 18 is displayed as 5 instead of 54 and the result of multiplying 2 by 71 is displayed as 12 instead of 142. If I multiply a positive one-digit number by a positive two-digit number on my calculator and it displays 26, list all possible number pairs I could have multiplied?

32. Find the 2016^{th} digit of number A which is formed by following pattern:
$A = 14916253649648110012 1\ldots$

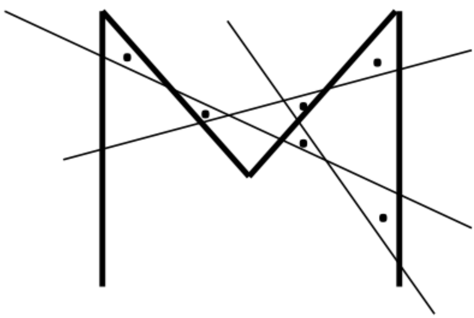
33. The diagram shows a right-angled triangle formed from three different coloured papers. The red and blue coloured papers are right-angled triangles, with the longest sides measuring 3 cm and 5 cm, respectively. The yellow paper is a square. Find the total area of the red and blue coloured papers.



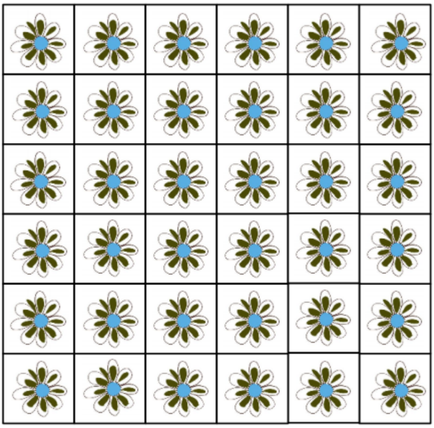
34. In the following figure, AC is a diameter of a circle. $\triangle ACB$ is an isosceles triangle with $\angle C = 90°$. D is a point on AB. Arc CD is part of a circle with centre B. If $AC = 10$cm, find the area of the shaded part. (Use $\pi = 3$).



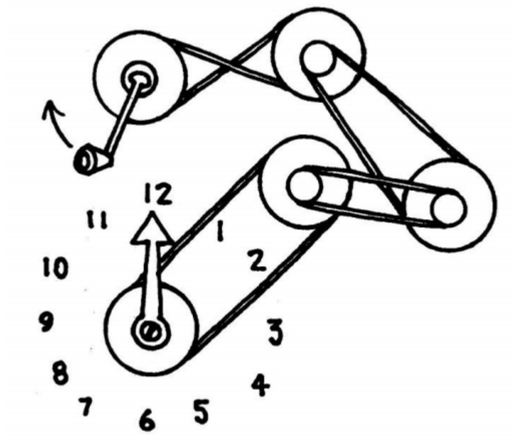
35. We have formed six triangles by drawing three straight lines on the M. That's not enough. Starting with a new M, let form nine triangles by drawing three straight lines.



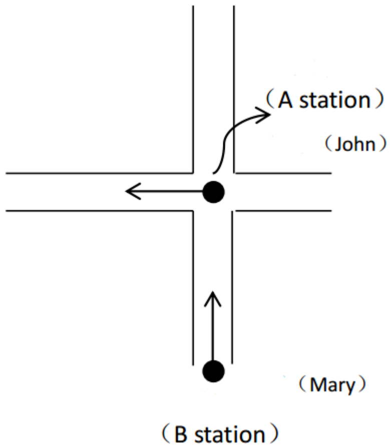
36. There are 36 flowers in the 6×6 boxes below. Please cut off 12 flowers from the boxes below so that each row and column contains the same amount of flowers.



37. The five identical wheels of this machine are connected by a series of belts. The outer rim of each wheel has a circumference of 8 centimetres. The rim of each wheel's inner shaft has a circumference of 4 centimetres. If the crank is rotated upwards one-quarter turn, what hour would the clock's hand point to?



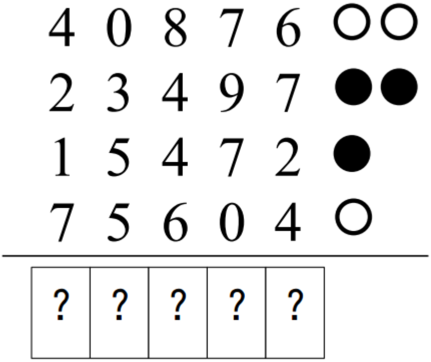
38. As shown below, the north-south and east-west highways are perpendicular to each other. One day, Mary drives to north from station B and John drives to west from station A. After 4 minutes, the distance of the two vehicles from the station A is the same. If they continue to travel in their respective directions, after 24 minutes, the two vehicles will still be the same distance from station A. The speed of John is 1.5kilometres per minute. Find the distance in kilometres between station A, and B.



39. The goal of this puzzle is to replace the question marks with a correct sequence of numbers. The black dots and white dots are the hints given to solve the question. The hints of the dots are stated as:

    (1) A black dot indicates that a number needed for the solution is in that row and in the correct position;

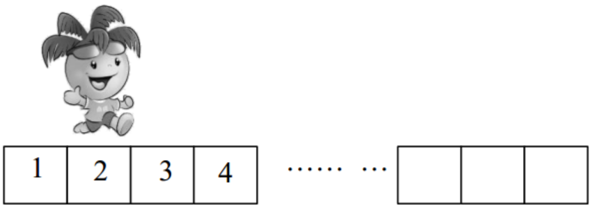
    (2) A white dot means that a number needed for the solution is in that row, but in the wrong position. Numbers do appear more than once in the solution, and the solution never begins with 0.



40. An evil dragon has three heads and three tails. You can slay it with the sword of knowledge, by chopping off all its heads and tails. With one stroke of the sword, you can chop off either one head, two heads, one tail, or two tails. But the dragon is not easy to slay! If you chop off one head, a new one grows in its place. If you chop off one tail, two new tails replace it. If you chop off two tails, one new head grows. If you chop off two heads, nothing grows. At least, how many chops do you need to slay the dragon?

41. Wendy has created a jumping game using a straight row of floor tiles that she has numbered $1, 2, 3, 4, \ldots$ Starting on tile 2, she jumps along the row, landing on every second tile, and stops on the second to last tile in the row. Starting from this tile, she turns

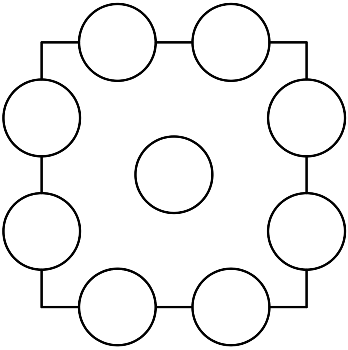
and jumps back toward the start, this time landing on every third tile. She stops on tile 1. Finally, she turns again and jumps along the row, landing on every fifth tile. This time, she stops on the second to last tile again. What is the at least minimum number of tiles?



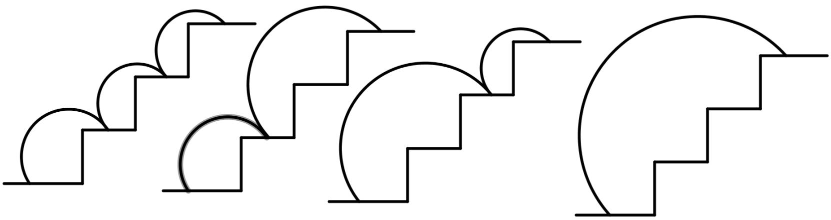
42. The “0” button on Ali’s calculator is broken, so he can not enter numbers which contain “0”. Unfortunately, his calculator does not display 0, even if it is part of an answer, either. So he can not enter the calculation 9×20 and does not attempt to do so. Also, the result of adding 56 and 24 is displayed as 8 (instead of 80) and the result of multiplying 7 by 29 is displayed as 23 (instead of 203). If Ali multiplies a single-digit number by a two-digit number on his calculator it displays 35. List all the possibilities for the two numbers that he could have multiplied.

43. Each number from 1 to 9 is placed, one per circle, into the pattern shown. The sums along each of the

four sides are equal. How many different numbers can be placed in the middle circle to satisfy these conditions?

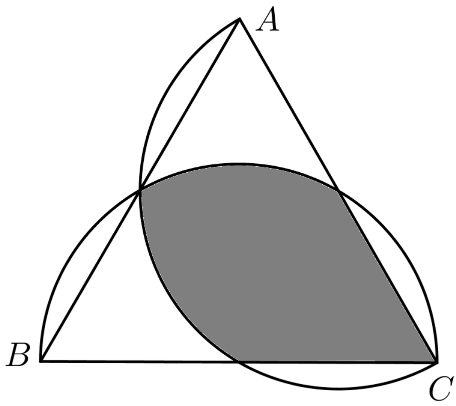


44. It is possible to climb three steps in exactly four different ways. In how many ways can you climb ten steps?



45. As shown in below figure, ABC is an equilateral triangle with a side length of 10cm. If using AC

and BC as radius to draw circles, what is the area of the shaded portion? (Use $\pi = 3.14$, and find an answer correct to 2 decimal places).



46. Stanley wrote a 4-digit number on a piece of paper and challenged Darrell to guess it. All the digits were different.
Darrell: It is 4607?
Stanley: Two of the numbers are correct but are in the wrong position.
Darrell: Could it be 1385?
Stanley: My answer is the same as before.
Darrell: How about 2879?
Stanley: Wow, two of the numbers are correct and in the right places as well.

Darrell: 5461?
Stanley: None of the digits is correct.
What was the number?

47. Four football teams A, B, C and D are in the same group. Each team plays 3 matches, one with each of the other 3 teams. The winner of each match gets 3 points; the loser gets 0 points; and if a match is a draw, each team gets 1 point. After all the matches, the results are as follows:

    (1) The total scores after the 3 matches for the four teams are consecutive odd numbers.

    (2) D has the highest total score.

    (3) A has exactly 2 draws, one of which is the match with C.

    Find the total score for each team.

48. Jane has 9 boxes and 9 accompanying keys. Each box can only be opened by one key. If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes.

49. Starting with the "1" in the centre, the spiral of consecutive integers continues, as shown. What is

the difference between the numbers that appears directly below and above 2016?

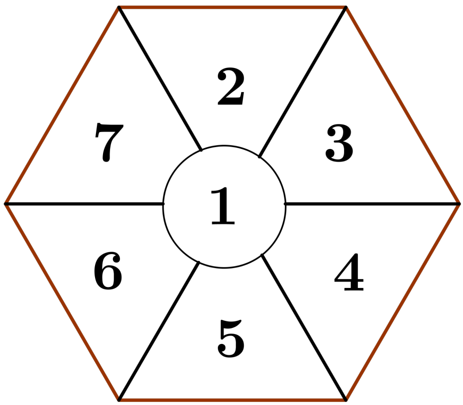
	13	14	15	16	17	
	12	3	4	5	18	
	11	2	1	6	19	
27	10	9	8	7	20	
26	25	24	23	22	21	

50. Let $\prod(\overline{abc}) = a \times b \times c$. For example, $\prod(137) = 1\times3\times7 = 21$ and $\prod(234) = 2\times3\times4 = 24$. Find the value of the expression $\prod(200)+\prod(201)+\prod(202)+\cdots+\prod(300)$.

51. A combination lock on a safe needs a 6-letter sequence to unlock it. This is made from the letters A, B, C, D, E, F with none of them being used twice. Here are three guesses at the combination
$C\ B\ A\ D\ F\ E$
$A\ E\ D\ C\ B\ F$
$E\ D\ F\ A\ C\ B$
In the FIRST guess only ONE letter is in its correct

place. In the SECOND guess only TWO letters are in their correct places and these two correct places are not next to each other. In the THIRD guess THREE letters are in their correct places. Each of the 6 letters is in its correct place once. What is the correct combination?

52. A flower plantation has 7 areas as shown in the figure. Plant them with flowers of 4 different colors so that each area has only one colour. In how many ways can we plant the flowers so that the neighbouring regions all have different colors?

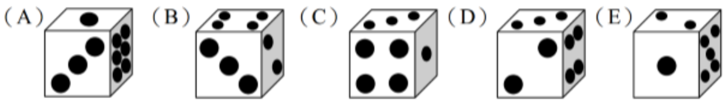


53. Know that $\dfrac{a}{b} = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{131}$ and $\dfrac{a}{b}$ is an irreducible fraction. Prove that a is composite number.

54. Numbers like 1001, 23432, 897798, 3456543 are known as palindromes. If all of the digits $2, 7, 0$ and 4 are used and each digit cannot be used more than twice, find the number of all different palindromes that can be formed.

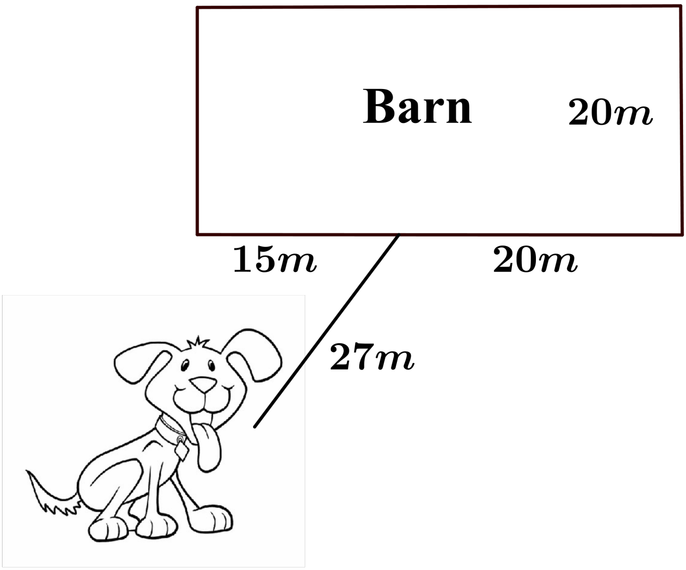
55. The product $1! \times 2! \times 3! \times \ldots \times 2015! \times 2016!$ is written on the blackboard. Which factor, in terms of factorial of an integer, should be erased so that the remaining product is the square of an integer number? (The factorial sign $n!$ stands for the product of all positive integers less than or equal to n.)

56. Each of the following five six-sided die has 1, 2, 3, 4, 5 and 6 spots on its faces. Which one has a different arrangement of spots from the other four?

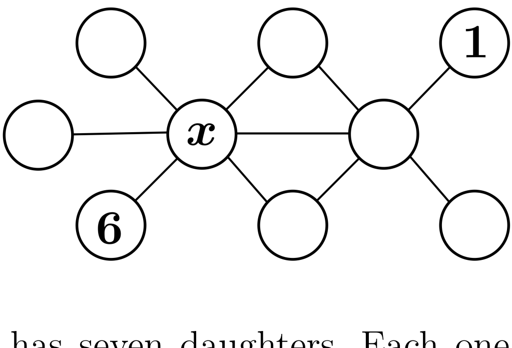


57. The diagram shows that if a rope is folded once and be cut in half, it will separate into 3 pieces; and if it is folded twice instead, it will separate into 5 pieces. If it is folded 6 times instead, into how many pieces will it separate?

58. A farmer fastens the end of his dog's leash to the edge of his barn at a point that is 15m from one corner and $25m$ from the other corner of the barn, as illustrated in the diagram below. The Barn is 20m wide and the leash is 27m long. Calculate the area the dog is able to reach while leashed to the wall, to the nearest whole square metre!



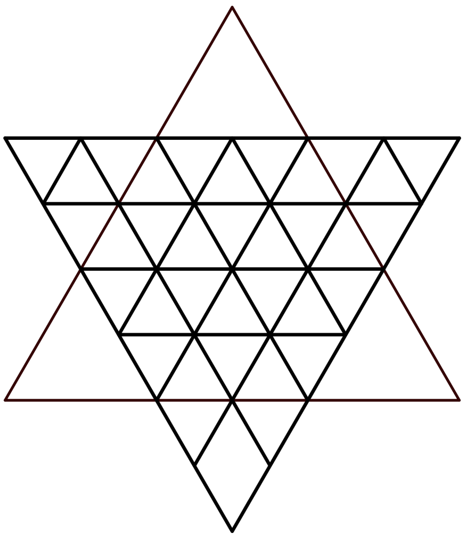
59. In the diagram, each of the integers 1 through 9 is to be placed in one circle so that the integers in every straight row of three joined circles add to 18. The 6 and 1 have been filled in. Find the value of x.



60. A family has seven daughters. Each one after the

eldest is two years younger than the one born before. If the eldest daughter is three times as old as the youngest, how old is the eldest?

61. How many triangles are there in below diagram?



62. The 100000 tickets for an event are numbered from 00000 to 99999. If a number contains two adjacent

digits which differ by exactly 5, it wins a door prize. How many door prizes will be needed if all tickets are sold?

63. 7 out of 8 coins are known to be real and have the same weight. The other one may also be real, but may be a fake coin, which is either heavier or lighter than a real coin. We want to know if there is a fake coin. If so, we wish to know whether it is heavier or lighter, but it is not necessary to identify the fake coin. What is the minimum number of weighing on a scale that would accomplish the task?

64. Find the value of

$$1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{999}{1234}}}}}}}?$$

65. Dates can be written as an 8-digit integer in the format $YYYYMMDD$. For example, 20160125 stands for January 25th 2016. How many days are there in

the year 2016 when the 8-digit representation contains equal numbers of the digits $0, 1, 2$?

66. From 2015 to 6999, how many integers have their sum of digits divisible by 5?

67. Find the sum of all numbers from 1 to 2000, the sum of the digits of which are even?

68. In the correct addition below, each letter stands for a digit. What is the value of the sum $A + 10B + C + D + E + F$?

$$\begin{array}{rccc} & A & 2 & E \\ & 1 & B & D \\ + & F & 2 & C \\ \hline & 6 & 3 & 2 \end{array}$$

69. The numbers $1, 2, \ldots, 25$ are to be placed in a 5×5 table, with one number exactly in each square. Consecutive numbers occupy squares with a common side. Three of the numbers have been placed, as shown in the diagram below. Find the number of different placements of the other 22 numbers.

19		13		
		1		

70. Tom and Jerry play the following game. Tom has some number of coins and Jerry has none. Jerry can take any (non-zero) number of coins from Tom. Then Tom can take some (again, non-zero) number of coins back, but necessarily a different number. Then again, Jerry takes some from Tom, but necessarily a number which did not occur before. And so on. The game stops when someone cannot make a move. What is the largest number of coins Jerry can have at the end if:

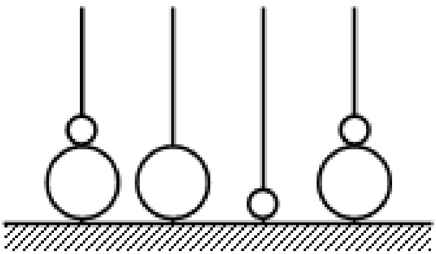
    (a) Tom had 13 coins at the beginning?

    (b) Tom had 50 coins at the beginning?

71. Filll the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into the boxes below so that the expression will produce the largest product. (Each digit can be used only once)

$$\square\square\square\square\square \times \square\square\square \times \square$$

72. There are 4 fixed steel pins, 3 big balls and plenty of small balls. Consider every small ball as the digit “3” and every big ball as the digit“5” (as shown in Fig. 6, it stands for 8538). Now, these balls are placed on the steel pins. Definitely all pins have balls. Start to read the numbers from left to right. (The sum of the balls representing numbers on every pin is less than 10.) How many different four-digit numbers can be read?

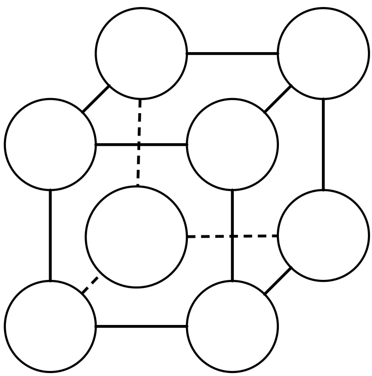


73. Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner. If the whole number is 29 and Peter is the first player, what perfect

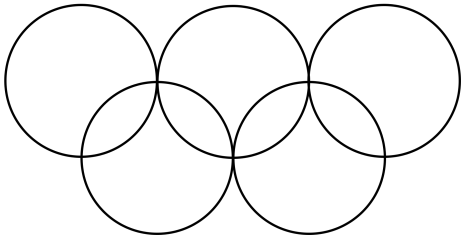
number must he subtract in order for him to definitely win. (Note: $4; 9$ and 16 are examples of perfect squares)

74. Five points lie on a line. Alex finds the distances between every possible pair of points. He obtains, in increasing order, 2, 5, 6, 8, 9, k, 15, 17, 20 and 22. What is the value of k?

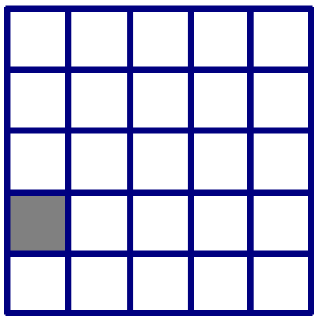
75. Eight of the digits $1, 2, 3, \ldots$ and 9 are arranged, one per circle with each circle on one of the eight edges of the cube, on the cube shown so that S, the sum of the numbers on each face of the cube, is the same. Seven of the eight numbers are labelled, but one is not. It is known that S is not divisible by the missing number. What is the missing number x?



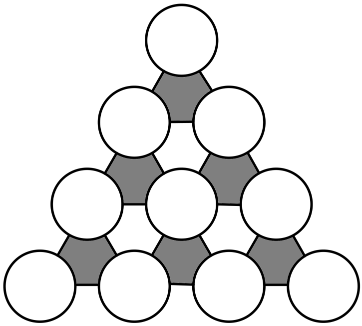
76. How many possible solutions are there in arranging the digits 1 to 9 into each closed area so that the sum of the digits inside every circle is the same. Each closed area contains only one digit and no digits are repeated. Draw all possible solutions.



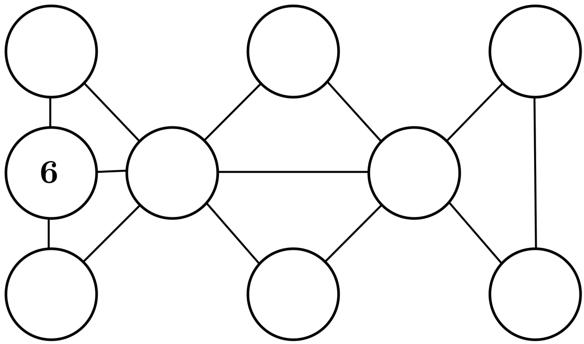
77. A frog is sitting on a square adjacent to a corner square of a 5×5 board. It hops from square to adjacent square, horizontally or vertically, but not diagonally. Prove that it cannot visit each square exactly once.



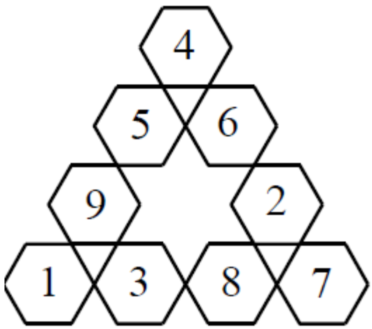
78. Place the numbers 0 through 9 in the circles in the diagram below without repetition, so that in each of the six small triangles pointing up (shaded triangles), the sum of the numbers in the vertices is the same.



79. Each of the nine circles in the diagram below contains a different positive integer. These integers are consecutive and the sum of the numbers on each of the seven lines is 23. The number in the circle at the top right corner is less than the number in the circle at the bottom right corner. Eight numbers have been erased. Restore them.



80. In the diagram below, the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are placed one inside each hexagon, so that the sum of the numbers along each of the three sides of the triangle is 19. If you are allowed to rearrange the numbers but still have to keep the sum along the sides equal (can be different from 19), what is the smallest possible sum and what is the largest possible sum?



81. Place the digits 1 to 6 in the grid so that no digit is repeated in a row, column or diagonal.

2			1		
					4
	2				
				6	
		5			1
3					

82. An cut the pizza into n equal slices and then she labelled them with numbers $1, 2, \ldots, n$ (she used each number exactly once). The numbering had the property that between each two slices with consecutive numbers (i and $i+1$) there was always the same number of other slices. Then came Binh the glutton and ate almost the whole pizza, leaving only the three neighbouring slices with the numbers $11, 4,$ and 17 (in this exact order) on them. How many slices did the pizza have?

83. In one of the lecture halls at Hanoi city the seats are arranged in a rectangular grid. During the lecture of

geometry there were exactly 11 boys in each row and exactly 3 girls in each column. Moreover, two seats were empty. What is the smallest possible number of the seats in the lecture hall?

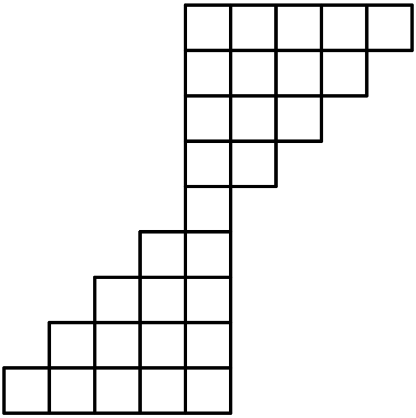
84. Vinh thought of three distinct positive integers a, b, c such that the sum of two of them was 800. When he wrote numbers a, b, c, $a+b-c$, $a+c-b$, $b+c-a$ and $a+b+c$ on a sheet of paper, he realized that all of them were primes. Determine the difference between the largest and the smallest numbers on Vinh's paper.

85. A polynomial $P(x)$ of degree 2015 with real coefficients such that $P(n) = 3^n$ for all $n = 0, 1, \ldots, 2015$. Evaluate $P(2016)$.

86. In an isosceles triangle ABC, fulfilling $AB = AC$ and $\angle BAC = 99.4°$, a point D is given such that $AD = DB$ and $\angle BAD = 19.7°$. Compute $\angle BDC$

87. There are 29 unit squares in the diagram below. A frog starts in one of the five (unit) squares in the bottom row. Each second, it jumps either to the square directly above its current position (if such a square exists), or to the square that is one above and one to the right from its current square (if such

a square exists). The frog jumps every second until it reaches the top. How many distinct paths can it take from the bottom to the top row?



88. If sides a, b, c of a triangle satisfy

$$\frac{3}{a+b+c} = \frac{1}{a+b} + \frac{1}{a+c},$$

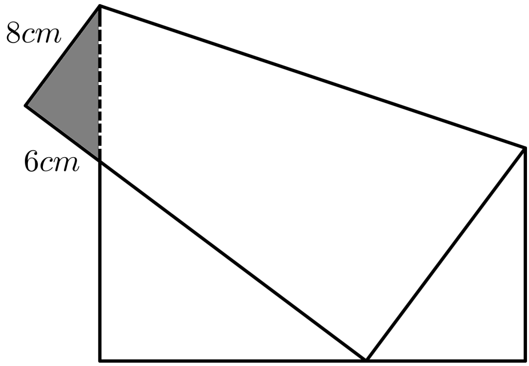
what is the angle between sides b and c?

89. Consider a number that starts with 122333444455555... and continues in such a way that we write each positive integer as many times as its value indicates. We stop after writing 2016 digits. What is the last digit of this number?

90. I chose two numbers from the set $\{1, 2, \ldots, 9\}$. Then I told An their product and Binh their sum. The following conversation ensued:
An: *"I don't know the numbers."*
Binh: *"I don't know the numbers."*
An: *"I don't know the numbers."*
Binh: *"I don't know the numbers."*
An: *"I don't know the numbers."*
Binh: *"I don't know the numbers."*
An: *"I don't know the numbers."*
Binh: *"I don't know the numbers."*
An: *"Now I know the numbers."*

What numbers did I choose?

91. A square sheet of paper is folded so that one of its vertices is precisely on one of the sides. As in the picture, there is a small triangle formed where the paper does not overlap. The length of its outer side that is adjacent to the line of the folding is 8 cm, and the length of the other outer side is 6 cm. What is the side length of the paper?



92. There are $n > 24$ women sitting around a great round table, each of whom either always lies or always tells the truth. Each woman claims the following:
    She is truthful.
    The person sitting twenty four seats to her right is a liar.
    Find the smallest n for which this is possible.

93. Ten people - five women and their husbands - took part in E events. We know that no married couple took part in the same event, every possible pair of non-married people (including same-sex pairs) took part in exactly one event together, and one person attended only two events. What is the smallest E for which this is possible?

94. Find n, the number of positive integers not exceeding 1000 such that the number $\lfloor \sqrt[3]{n} \rfloor$ is a divisor of n. Note: The symbol $\lfloor x \rfloor$denotes the integral part of x, i.e. the greatest integer not exceeding x.

95. A sequence (a_n) is given by $a_1 = 1$, and $a_n = \lfloor \sqrt{a_1 + a_2 + \cdots + a_{n-1}} \rfloor$ for $n > 1$. Determine a_{1000}. Note: The symbol $\lfloor x \rfloor$ denotes the integral part of x, i.e. the greatest integer not exceeding x.

96. Let (α, β) be an open interval, with $\beta - \alpha = \dfrac{1}{2016}$. Determine the maximum number of irreducible fractions $\dfrac{a}{b} \in (\alpha, \beta)$ with $1 \leq b \leq 2016$?

97. Let $p = \overline{abc}$ be a three-digit prime number. Prove that the equation $ax^2 + bx + c = 0$ has no rational roots.

98. How many integers belong to $(a, 2016a)$, where a $(a > 0)$ is a given real number?

99. Given an array of number $A = \{672; 673; \ldots; 2016\}$ on table. Three arbitrary numbers $a, b, c \in A$ are step by step replaced by number $\dfrac{1}{3}\min\{a, b, c\}$. After 672 times, on the table there is only one number m. Prove that $m < 1$.

100. Let n be a positive integer and $P(n)$ the product of the non-zero digits of n. Find the largest prime divisor of the number

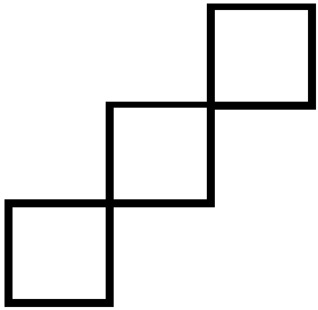
$$P(1) + P(2) + P(3) + \cdots + P(999).$$

101. There is a group of 30 people where everyone is familiar with at least 25 others. Prove that there exists a group of at least 6 people who know each other. Would this hold true for 7 people?

102. A 13×13 checkerboard's middle square is missing. Prove that the board cannot be paved with 1×4 rectangles (there can be no overlap).

103. There is a 5×5 checkerboard filled by white or black squares. Prove that there exist four unit squares of the same colour that are at the intersection of two columns and two rows.

104. What is the largest number of below shape can you cut from an 8×8 checkerboard?
"Note: each shape covers exactly three unit squares of the checkerboad".



105. Is it possible to build a $8 \times 8 \times 9$ cuboid from 32 pieces of smaller $2 \times 3 \times 3$ cuboids?

106. A $6 \times 6 \times 6$ cube consists of 216 small $1 \times 1 \times 1$ cubes. In how many ways can we pair two small cubes so that they have at most 2 vertices in common?

107. We know that any triangle can be cut into four smaller congruent triangles. On the other hand, can we cut any triangle into four similar, but not all congruent triangles?

108. Let us choose arbitrarily n vertices of a regular $2n-gon$ and colour them red. Remaining vertices are coloured blue. We arrange all red-red distances into a nondecreasing sequence and do the same with blue-blue distances. Prove that the sequences are equal.

Xin chân thành cám ơn sự quan tâm và những ý kiến đóng góp của bạn đọc!